Momentum in Best of Three Contests

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Motivation

Feedback of intermediate outcomes in dynamic contests can change the behavior of contestants during the contest.

In multi-round contests, winning a round may increase the probability of winning subsequent rounds.

- This correlation between winning adjacent rounds is referred to as momentum

Two sources of momentum are often cited in the literature

- Strategic Momentum - position in contest
- Psychological Momentum - "winner's effect"

Motivation

The source of strategic momentum has been formalized

- It arises endogenously in best of n contests when one contestant is ahead of the other

Harris and Vickers (1987), Konrad and Kovenock (2009)

Identifying psychological momentum empirically

- Correlation between winning subsequent rounds when strategic momentum is known non-factor

Dechenaux et al (2015), Mago et al. (2013)

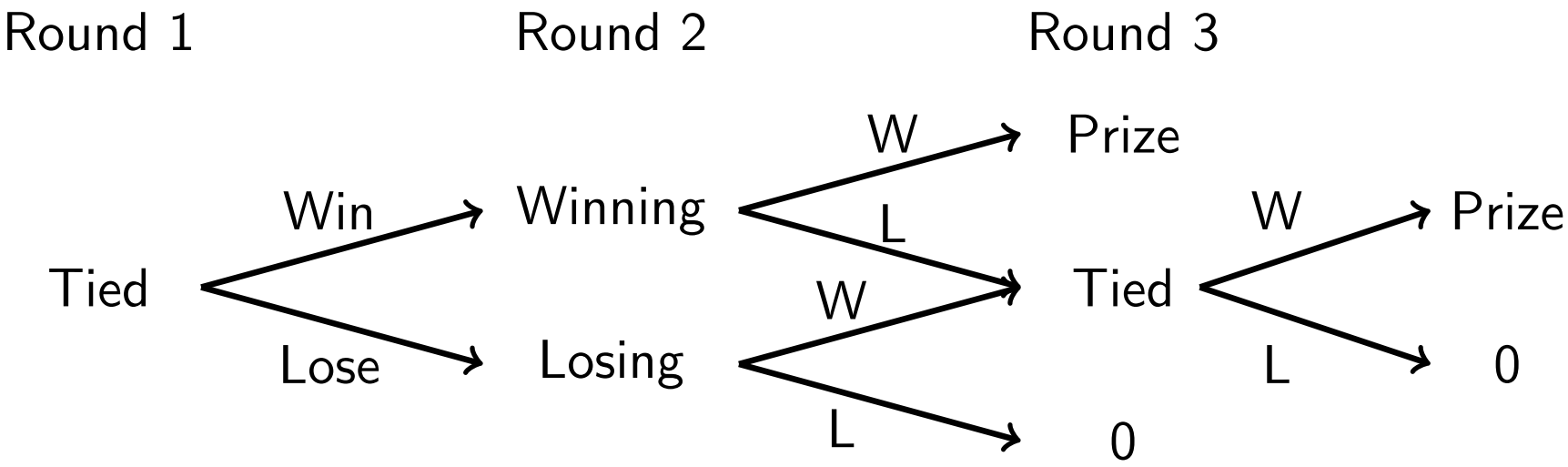
Psychological momentum

In this paper, we endogenize psychological momentum in a best of three contest

- Incomplete information: contestants unsure of ability relative to opponent
- Contestants learn relative ability from contest feedback

Motivating example

Best of three contest, strategic momentum.



Effort choice in each round depends on prize of winning that round

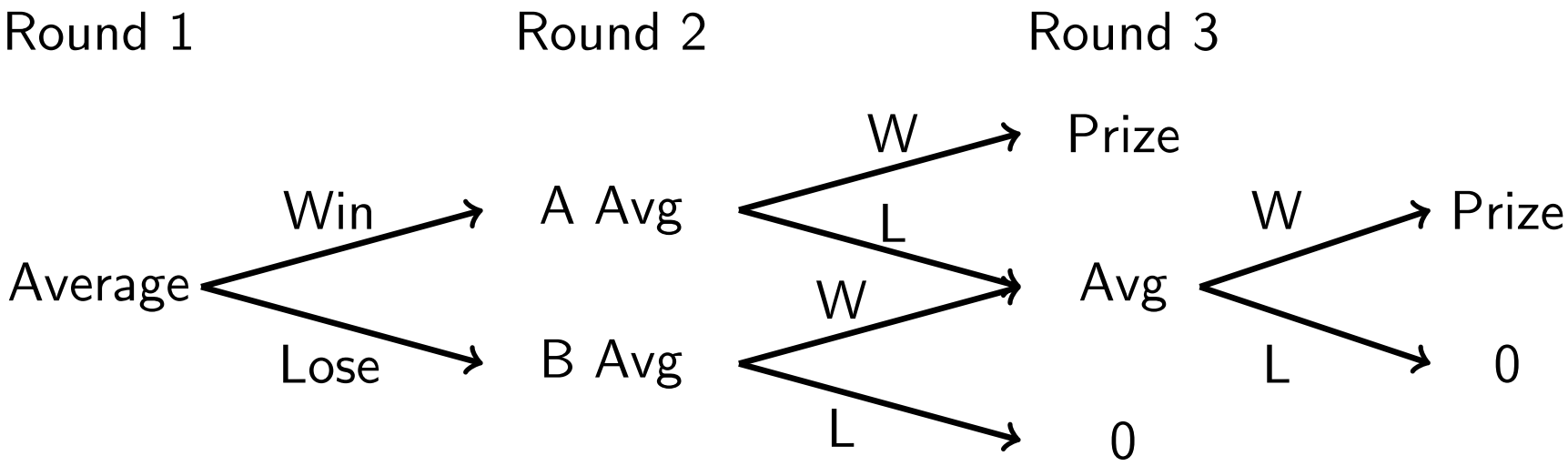
- Prize of a round is endogenous (except round 3)

Strategic momentum differentiates effort choices in round 2

- Round 2's prize is higher for winning contestant

Motivating example

Best of three contest, psychological momentum.



Effort choice in each round depends on expected relative ability

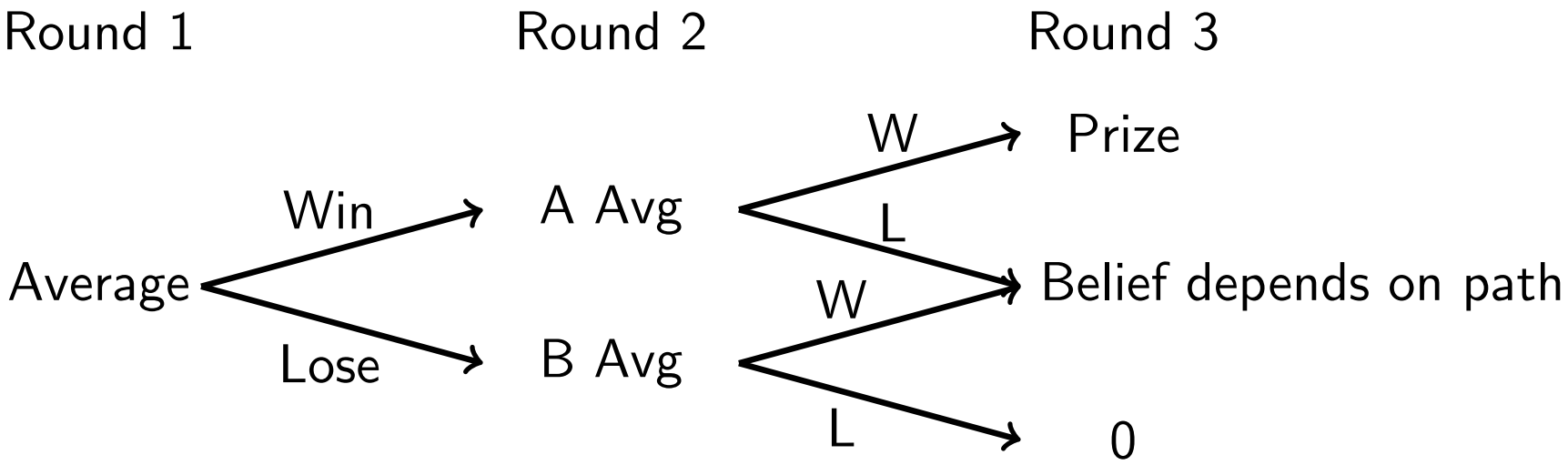
- Expected ability depends on past outcomes and effort choices

Psychological momentum differentiates effort choices in round 2

- Winning contestant is more optimistic about ability then losing contestant

Motivating example

Best of three contest, psychological momentum.



Psychological momentum differentiates effort choices in round 3

- This can create correlation in outcomes in round 2 and 3 which is not present when only considering strategic momentum

Preview of results

Simple simulations suggest positive momentum in the third round

- Contestant that wins the second round is more likely to win the third round
- Momentum larger in contests with less randomness

Compared to baseline of complete information about ability:

- Momentum in both the second and third period is larger
- Effort choice is higher in the first round, about the same in second round

Model

Two ex ante symmetric contestants, i and j, independently endowed with ability, $a \in \{1, a_h\}$

- Endowment is unobserved by either contestant, information is symmetric
- Common knowledge that each of four states is equally likely
- Ability is fixed for duration of contest

Symmetric prize V^* awarded to contestant that wins two rounds

- No intermediate prizes
- Third round only played if necessary

Stage Game

Contestants choose effort level, e, in each round, r.

- Cost of effort $c(e) = ke^{\alpha}$ with $k > 0, \alpha > 1$.
- Output $x(a, e) = ae$

Contest success function: tournament with uniform noise parameterized by m

$$\Pr(w^i | x^i, x^j) = \frac{x^i - x^j + m}{2m}$$

Payoffs for player i,

$$\pi^i(x^i, x^j) = \text{Winner's Prize} * \Pr(w^i | x^i, x^j) + \text{Loser's Prize} * \Pr(w^j | x^i, x^j) - c(e^i)$$

Information and Strategies

Contestants observe own effort choice and outcome of previous rounds

- History at round $r \geq 2$ for contestant i

$$h_r^i = ((e_1^i, w_1), \ldots, (e_{r-1}^i, w_{r-1}))$$

- Strategy for contestant i is an effort choice at each history of play, $\sigma^i : H^i \to \mathbb{R}^+$, $\sigma^i(h_r^i) = e_r^i$.
- Conjecture about the opponent's strategy is $\tilde{\sigma}^j$

Beliefs about ability are determined by history and conjecture

- Probability high ability: $\theta_r^i(h_r^i, \tilde{\sigma}^j)$, $\tilde{\theta}_r^j(h_r^i, \tilde{\sigma}^j)$
- Expected abilities: $\mathbb{E}[a^i | h_r^i, \tilde{\sigma}^j]$, $\mathbb{E}[a^j | h_r^i, \tilde{\sigma}^j]$

Equilibrium

We will focus attention on symmetric pure strategy BNE. An equilibrium is a pair of strategies (σ^i, σ^j) such that

- Beliefs are updated via Baye's rule whenever possible

$$\theta_r^i(h_r^i, \tilde{\sigma}^j) = \Pr(a^i = a_h | h_r^i, \tilde{\sigma}^j) \text{ and } \tilde{\theta}_r^j(h_r^i, \tilde{\sigma}^j) = \Pr(a^j = a_h | h_r^i, \tilde{\sigma}^j).$$

- Effort choices are optimal given conjectures and beliefs for every history

$$\sigma^i(h_r^i) \in \arg\max_{e_r^i} \pi_r^i(e_r^i, \tilde{e}_r^j, \theta_r^i, \tilde{\theta}_r^j).$$

- Conjectures of opponent's strategies match true strategies, $\tilde{\sigma}^j = \sigma^j$ and $\tilde{\sigma}^i = \sigma^i$, and strategies are symmetric $\sigma^i = \sigma^j$.

Round Continuation Value

The value of entering round r with history h_r^i and conjecture $\tilde{\sigma}^j$:

$$V_r^i(h_r^i, \tilde{\sigma}^j) = \max_{e_r^i} \pi_r^i(e_r^i, \tilde{e}_r^j, \theta_r^i, \tilde{\theta}_r^j)$$

When an outcome in round r does not end the contest, the value of the outcome to the contestant is endogenous.

$$\begin{aligned}\pi_r^i(e_r^i, \tilde{e}_r^j, \theta_r^i, \tilde{\theta}_r^j) = {} & V_{r+1}^i(h_r^i \cup (e_r^i, w_r^i), \tilde{\sigma}^j) \Pr(w_r^i | e_r^i, \tilde{e}_r^j, \theta_r^i, \tilde{\theta}_r^j) \\ & + V_{r+1}^i(h_r^i \cup (e_r^i, w_r^j), \tilde{\sigma}^j) \Pr(w_r^j | e_r^i, \tilde{e}_r^j, \theta_r^i, \tilde{\theta}_r^j) - c(e_r^i)\end{aligned}$$

Endogenous prize from winning round r

$$V_r^{i*}(h_r^i, \tilde{\sigma}^j, e_r^i) = V_{r+1}^i(h_r^i \cup (e_r^i, w_r^i), \tilde{\sigma}^j) - V_{r+1}^i(h_r^i \cup (e_r^i, w_r^j), \tilde{\sigma}^j)$$

Effort Choice

Optimal effort level is

$$c'(e_r^i) = \frac{V_r^{i*}(h_r^i, \tilde{\sigma}^j, e_r^i)}{2m} \mathbb{E}[a^i | h_r^i, \tilde{\sigma}^j] + \text{2nd order effect}$$

- Strategic effect: $V_r^{i*}(h_r^i, \tilde{\sigma}^j)$.
- Psychological effect: $\mathbb{E}[a^i | h_r^i, \tilde{\sigma}^j]$.
- Effect of an increase in effort choice on value of future rounds: 2nd order effect.

Momentum in Round 2

Proposition 1

The contestant that wins round one will exhibit both strategic and psychological momentum in the second round.

Psychological momentum: for any effort choice in round one winning player has higher expected ability.

Strategic Momentum:

- Prize of winning contest two for winning contestant:

$$V_2^{i*}(h_2^i, \tilde{\sigma}^j, e_2^i) = V^* - V_3^i(h_3^i, \tilde{\sigma}^j)$$

- Prize of winning contest two for losing contestant:

$$V_2^{j*}(h_2^j, \tilde{\sigma}^i, e_2^j) = V_3^j(h_3^j, \tilde{\sigma}^i) - 0$$

- Third round: $V^* > V_3^i(h_3^i, \tilde{\sigma}^j) + V_3^j(h_3^j, \tilde{\sigma}^i)$

Momentum in Round 3

There is no strategic momentum in the third round.

Psychological momentum arises from differing beliefs: $\mathbb{E}[a^i|h_r^i, \sigma^j]$ vs. $\mathbb{E}[a^j|h_r^j, \sigma^i]$

Contestant i won round one but lost round two

- If first period effort is large compared to second period effort levels, first round is more informative.

Contestant j lost round one but won round two

- Overcame momentum of opposing player and won despite less effort. First round outcome more easily attributed to luck.

Simulations

We conduct simulations with $c(e) = e^2$, $a_h = 2$, $V^* = 1$ while varying randomness in the determination of the winner.

Recall: probability success function ($x = ea$)

$$\Pr(w^i | x^i, x^j) = \frac{x^i - x^j + m}{2m}$$

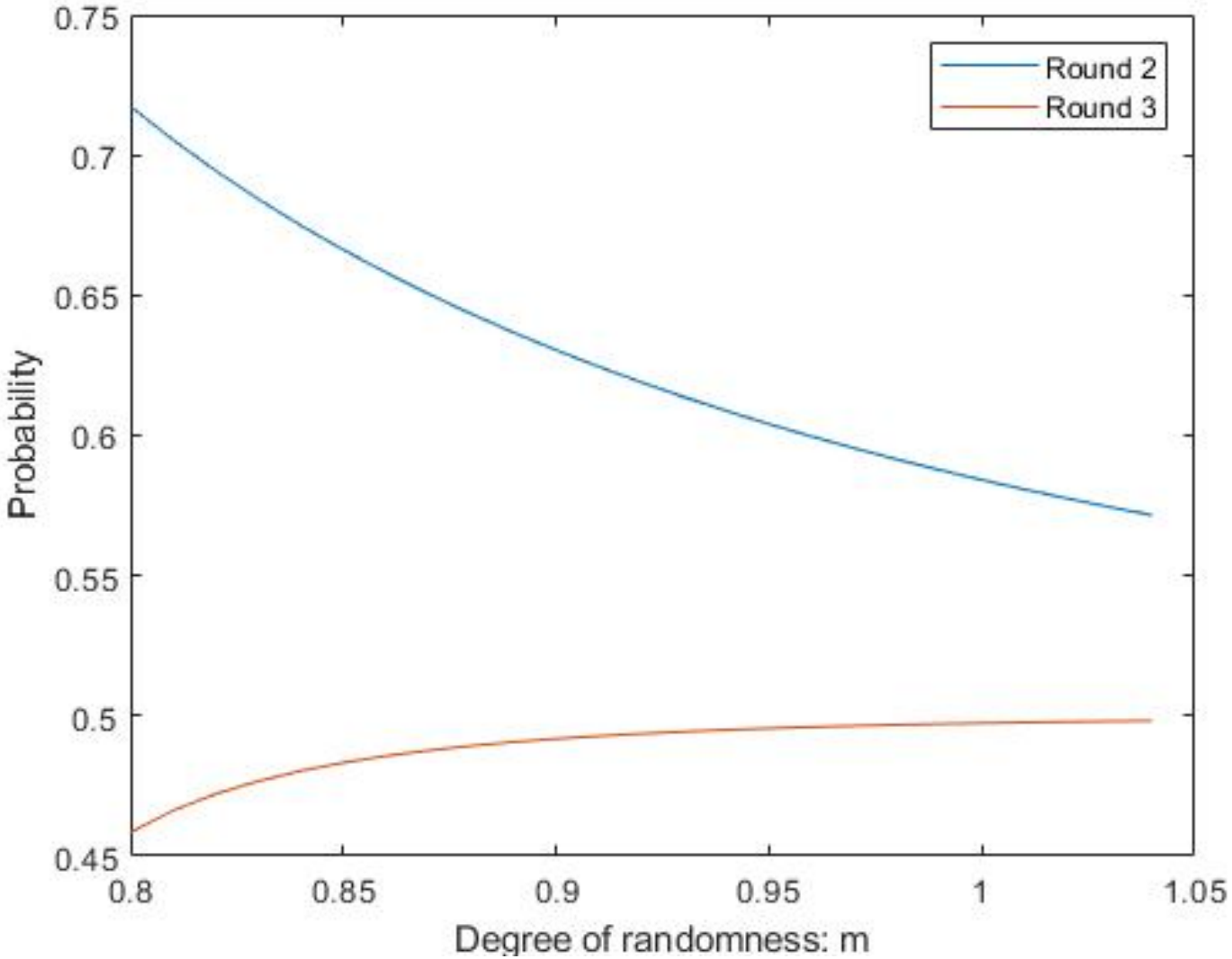
When randomness is large

- Lower efforts and less learning
- Less momentum in the second round, (almost) no momentum in third round

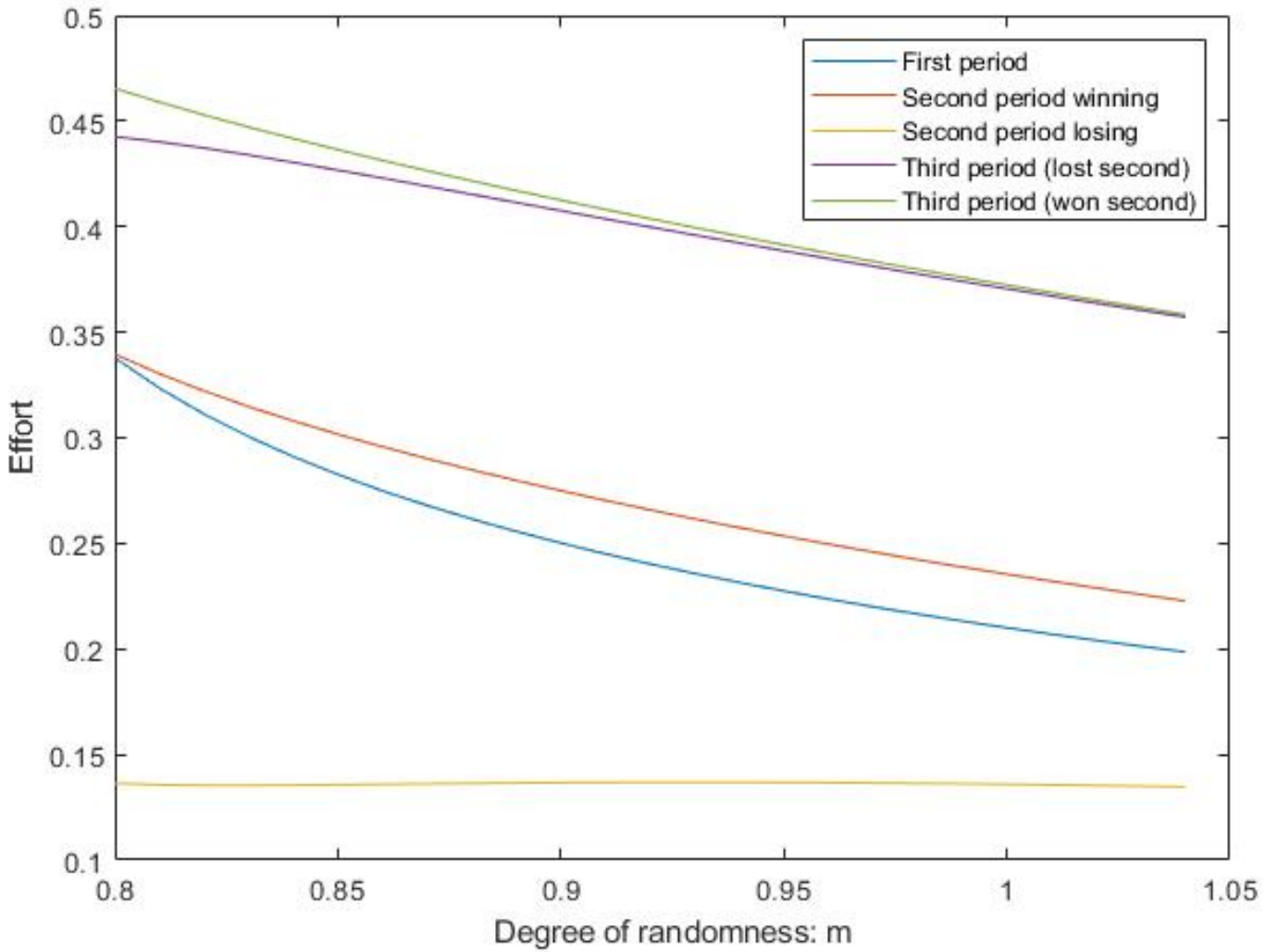
When randomness is small

- Higher efforts, more learning, more momentum in second round
- Momentum in third round for contestant who won second

Probability the winner of round 1 wins subsequent rounds



Equilibrium effort



Second order effects

The symmetric first order condition of the first round

$$c'(\hat{e}_1^i) = \frac{V_2^i((w_1^i, \hat{e}_1^i), \tilde{\sigma}^j) - V_2^i((w_1^j, \hat{e}_1^i), \tilde{\sigma}^j)}{2m} \mathbb{E}[a^i | \tilde{\sigma}^j] + \frac{1}{2}\left(\frac{\partial V_2^i((w_1^i, \hat{e}_1^i), \tilde{\sigma}^j)}{\partial \hat{e}_1^i} + \frac{\partial V_2^i((w_1^j, \hat{e}_1^i), \tilde{\sigma}^j)}{\partial \hat{e}_1^i} \right)$$

Marginal increase in effort does not affect the winning and losing player symmetrically

- Given win: increases belief about ability of both players
- Given loss: decreases belief about own ability, increases belief about opponent

Second order effects in the second round

The first order condition for the winning contestant is

$$c'(\hat{e}_2^i) = \frac{V^* - V_3^i(h_3^i, \tilde{\sigma}^j)}{2m}\mathbb{E}[a^i|h_2^i, \tilde{\sigma}^j] + \frac{\partial V_3^i(h_3^i, \tilde{\sigma}^j)}{\partial \hat{e}_2^i}\Pr(w_1^i, w_2^j|\hat{e}_1, \hat{e}_2^i, \tilde{\sigma}^j)$$

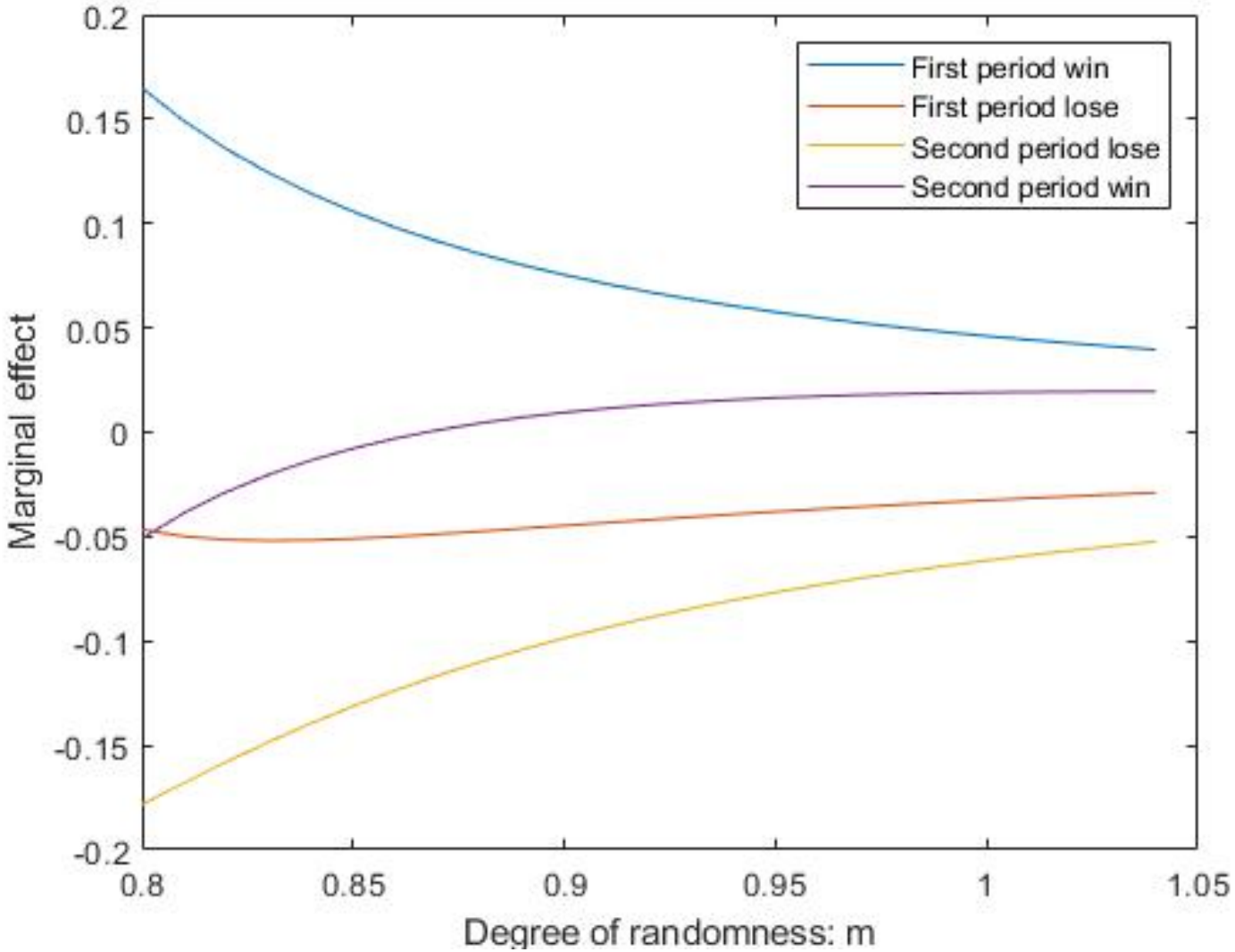
The first order condition of the losing contestant is

$$c'(\hat{e}_2^j) = \frac{V_3^j(h_3^j, \tilde{\sigma}^i)}{2m}\mathbb{E}[a^j|h_2^j, \tilde{\sigma}^i] + \frac{\partial V_3^j(h_3^j, \tilde{\sigma}^i)}{\partial \hat{e}_2^j}\Pr(w_1^i, w_2^j|\hat{e}_1, \hat{e}_2^j, \tilde{\sigma}^i)$$

Second order effects are ordered

$$\frac{\partial V_3^i(h_3^i, \tilde{\sigma}^j)}{\partial \hat{e}_2^i} < \frac{\partial V_3^j(h_3^j, \tilde{\sigma}^i)}{\partial \hat{e}_2^j} \gtrless 0$$

Effect of increase in effort on value of following round



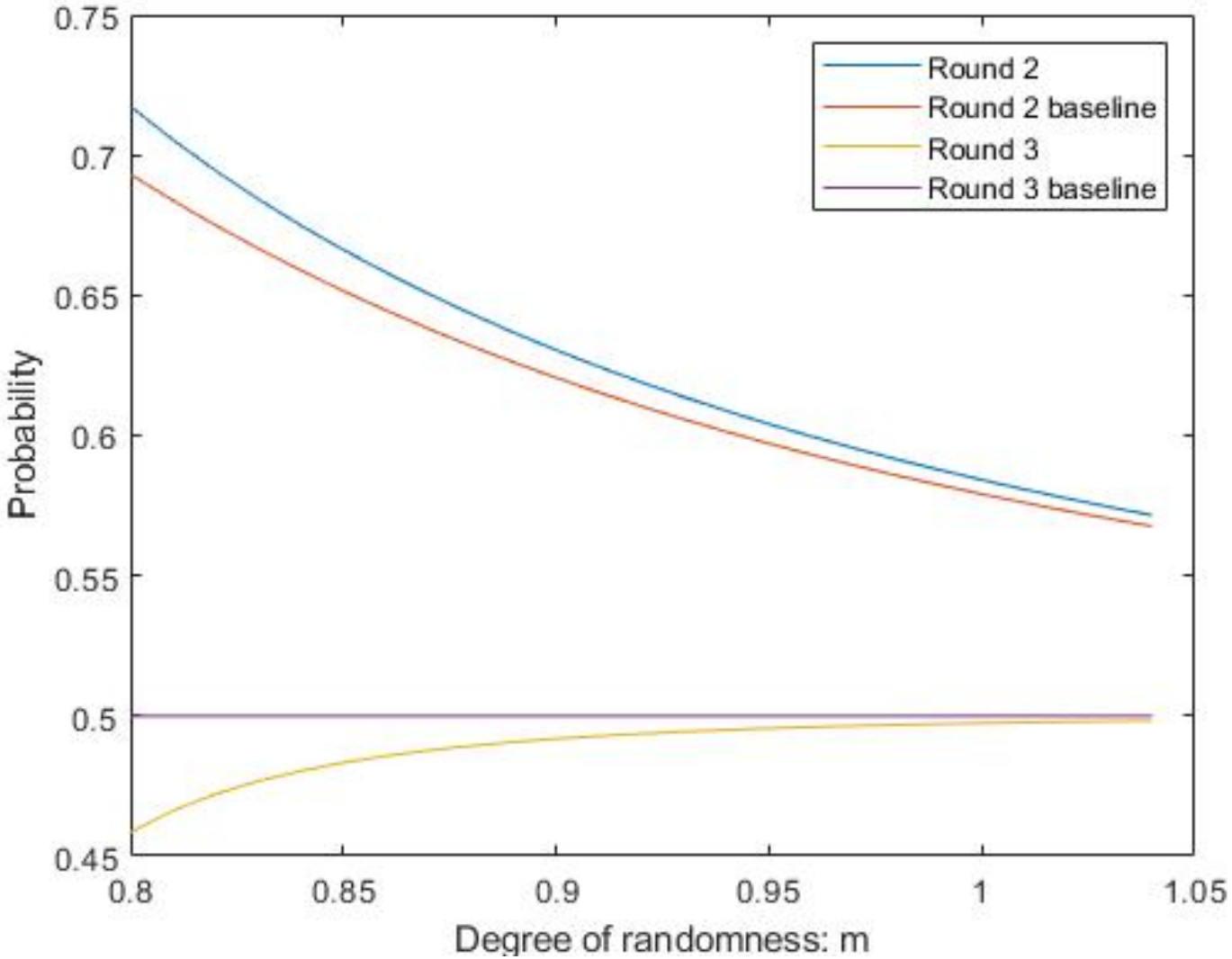
Baseline Model

Complete information about ability of each contestant:

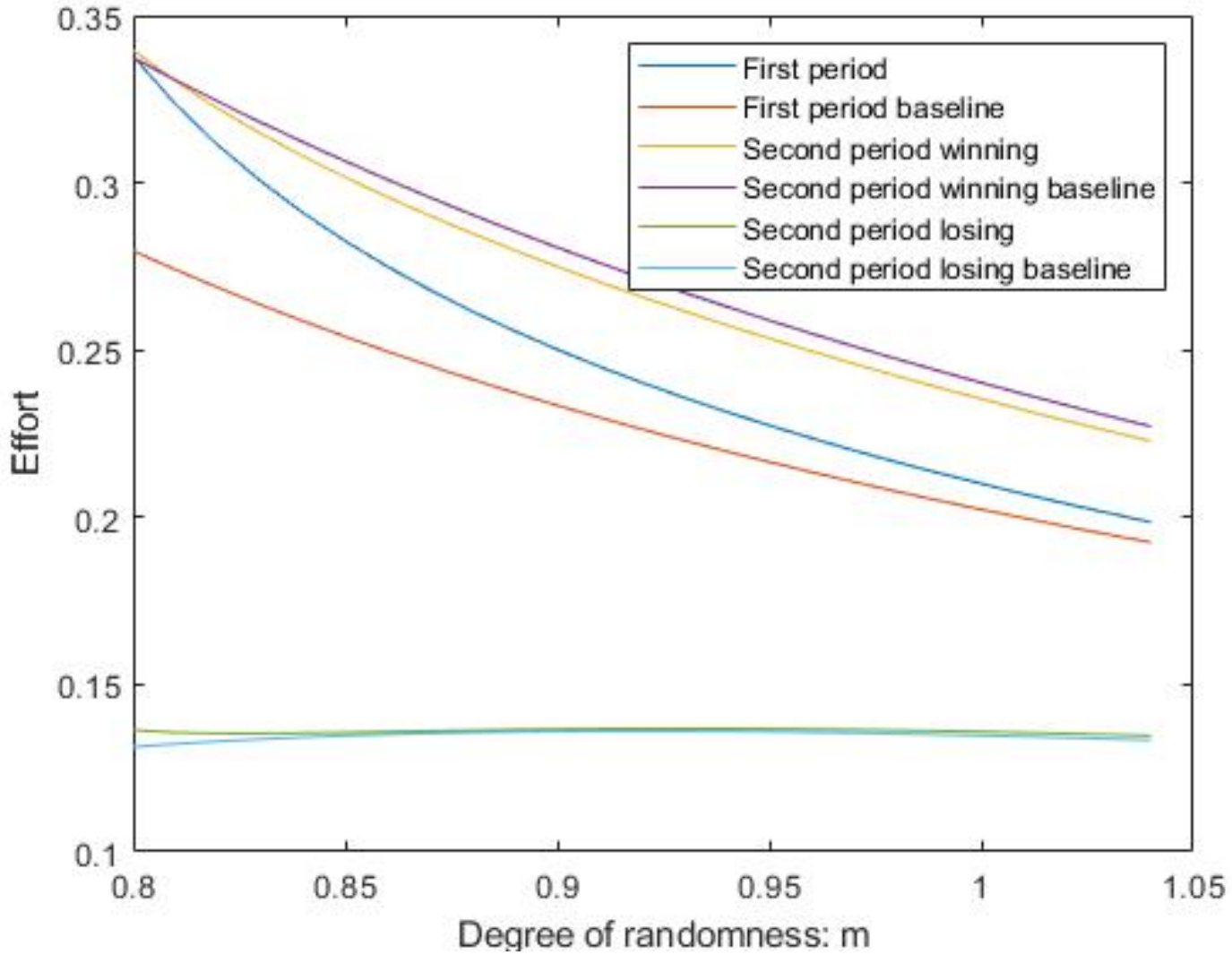
$$a_b^i = a_b^j = \mathbb{E}[a^i] = \frac{a_h + 1}{2}$$

- Psychological momentum no longer present as there is nothing to learn about relative ability.
- Current effort level does not impact value of future rounds.
- No marginal effects of future value functions in first order conditions.

Win probabilities compared to baseline



Effort in first and second round compared to baseline



Conclusion

Simple simulations suggest (some) momentum in the third round

- Contestant that wins the second round is more likely to win the third round
- Second contest is more informative than first about ability of the contestants
- Momentum larger in contests with less randomness

Second order effects have asymmetric impact on future beliefs

- Belief about opponent's ability always increases
- Belief about own ability depends on outcome of round

Compared to baseline

- Momentum in both the second and third period is larger
- Effort choice is higher in the first round, about the same in second round