Biased Contest Judges

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Abstract

We consider the design of a repeated contest in the presence of a potentially biased judge. Beliefs about the judge's bias may discourage contestants from exerting effort. In a repeated contest, the identity of a previous winner influences these beliefs. The contest designer is able to commit to a strategy of selectively and stochastically overruling the judge's ranking of the contestants. Overruling the judge can increase contestants' total expected effort and the designer's optimal intervention may improve or worsen expected outcomes for a type of contestant the judge is more likely to favor.

Keywords— Biased contest, Contest design, Feedback, Discouragement

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1 Introduction

Contests are frequently used to incentivize effort by rewarding contestants that outperform their peers. The designers of these contests often rely on a third party to evaluate the contestants: executives elicit nominations from employees' direct supervisors for performance bonuses and promotions, expert reviewers advise governments about which researchers to fund, and universities use students' teaching evaluations to give awards. However, these judges may be biased against certain types of contestants. Beliefs about the judge's bias evolve as contestants observe the results of previous contests and can reduce contestants' incentives to exert effort. In this paper, we study how a contest designer should use the recommendations of a potentially biased judge to incentivize effort provision over sequential contests.

Consider a firm that hires interns each year to perform basic tasks for the firm. At the conclusion of the internship, one of the interns is selected to stay on with the firm permanently. A manager that works directly with the interns and observes their efforts provides a recommendation to the firm about which intern should be selected. A successful internship program incentivizes the interns to work hard for the permanent position. Given the potential for the manager to be biased toward interns with an observable characteristic, e.g. gender, race, affiliation, age, etc., the identities of previous successful interns impact the beliefs that future interns have about their likelihood of earning the permanent position. A manager that is perceived to be biased towards one type of intern can undermine the ability of the program to incentivize hard work. Therefore, it may be in the interest of the firm to manage beliefs about bias in the program by including selection criteria beyond the rankings of the manager.

As an example, the firm may use a selection process that incorporates the ranking of the manager along with input from a committee that is less informed of the efforts of the interns. Moreover, the makeup of this committee can change from year to year, so that deviations from the manager's rankings appear random from one year to the next. The presence of the committee's input on the selection process adds variation in selection that is not directly related to effort. This directly reduces the incentives to exert effort in the current round of the program. However, this variation also limits the perception of bias in the selection process, potentially increasing the incentives for future interns to exert effort in the following year.

We model this situation as a repeated contest in which the contest designer cannot directly evaluate the effort of the contestants that participant in each contest. Instead, in each contest, a long-lived judge observes the effort of the contestants and privately ranks contestants for the designer. The designer observes the ranking provided by the judge and selects the winner of the contest. Winners of previous contests are publicly known. Importantly, contestants have observable characteristics that do not directly affect the designer's welfare, but the judge's ranking may be determined both by the contestants' efforts and their characteristics. How contestants' observable characteristics influence the judge's ranking is initially unknown to both the contestants and the contest designer, who share a common prior about the judge's bias.

Our analysis focuses on how the designer should use the ranking provided by the judge to maximize contestants' efforts. The designer can decide to respect the ranking of the judge and declare the highest ranked contestant the winner, or the designer can overrule the judge by choosing a different winner. More generally, for a given ranking, the designer can determine winning probabilities for each contestant. If the winning probabilities are independent of the ranking, i.e. the designer ignores the rankings of the judge, then the effort of the contestants cannot impact the identity of the winner. On the other hand, if the designer always follows the ranking of the judge by setting the winning probability of the highest ranked contestant to one, then the identities of the winners can quickly create a perception of a biased judge, even in the case where the judge is unbiased.

Contestants are incentivized to exert effort when higher effort increases their chance of winning the contest. Because the judge's ranking is informative about contestants' efforts, awarding the prize on the basis of this ranking incentivizes contestants to put forth effort. However, when contestants believe that the judge is biased against them and they are unlikely to win regardless of their effort choice, they put forth less effort. Since the ranking is also informative about the judge's bias, respecting the judge's ranking in the current contest can increase the perception of the judge's bias in future contests. This is the central trade-off that the designer faces: overruling the judge jams the signal about the judge's bias and leads to less pessimistic beliefs about the judge in future contests, but it does so at the cost of incentives in the current contest.

We find that the designer should always follow the ranking of the judge when there are no future rounds of the contest. This is true both in a single-period contest and in the final round of a repeated contest. In these cases, overruling the judge weakens incentives in that period, and with no continuation game, there are no benefits to changing contestants' beliefs. When there is a future round of the contest, the designer can increase the total expected effort by occasionally overruling the ranking of the judge for some initial beliefs about the judge's bias. Specifically, expected effort can be increased when there is a moderate likelihood that the judge is biased. When the judge is likely unbiased, there is no benefit of overturning the judge as contestants always exert effort. When the judge is very likely to be biased toward one type of contestant, other types never put in effort.

Intervention by the contest designer can increase total expected effort via two different mechanisms. First, if one outcome of a contest leads to contestants exerting more effort in the following contest, the designer can selectively overturn the judge's rankings to increase the likelihood of this outcome. Consider the case where the judge is likely to favor male candidates. If a female candidate loses the first contest, other female candidates may be discouraged from exerting effort in the following contest. Therefore, by sometimes overruling the judge when a man is ranked over a woman in the first contest, the designer increases the chance that the female candidate in the second contest will exert effort. This affirmative action style policy benefits the contestant that is less likely to be favored.

Second, overturning the judge's rankings jams the signal of the contest outcome, de-coupling the identity of the contest winner from the bias of the judge. This mechanism can be complementary to the affirmative action style policy. Reconsider the case with a judge that is likely to favor male candidates, but suppose that there is also a chance that the judge favors female candidates instead. By sometimes overruling the judge when the male candidate is ranked first, future male candidates can attribute previous losses to randomness beyond the judge's control and still decide to exert effort. In general, however, this signal jamming intervention does not necessarily benefit the disadvantaged type of contestant. We show that for certain prior beliefs about the judge's bias, the designer maximizes total effort by jamming the signal about the judge's bias created from the female contestant losing. The most effective way to jam this signal is to increase the likelihood the loss came from the designer's intervention rather than the judge's bias. This implies the designer must overturn the judge when the female contestant is ranked higher. While this intervention encourages a female contestant to exert effort in the following contest, it decreases the chance the female contestant wins the current contest.

In addition to designers using their contests to induce effort, they use contests to screen contestant ability. For example, firms often use internship programs not only as a source of cheap labor, but also as a mechanism to identify high ability candidates for permanent positions. While this incentive is not present in our main model, we consider an extension that allows the designer to use contests as a screening device. We show that while the benefit of overturning the judge is the same, the cost is higher as the judge's rankings are informative about the relative ability of the contestants. Despite this, the designer can still increase payoffs by overturning the judge for some prior beliefs about the judge's bias.

The paper adds to the literature on the impact of bias on contestants' effort. In this paper, a higher likelihood of bias unambiguously decreases effort. This is consistent with the idea that an unbiased contest leads to the most effort from contestants on average. In particular, if contestants are asymmetric then it is often optimal to favor the weaker contestant, leveling the competition and increasing overall effort (Baye et al., 1993; Che and Gale, 2003; Terwiesch and Xu, 2008). More recently it has been shown that bias can increase total effort even when contestants are ex-ante symmetric (Franke et al., 2013; Drugov and Ryvkin, 2017; Barbieri and Serena, 2020; Fu and Wu, 2020).[1] Given the contest success function in the current paper, the designer prefers to minimize the perceived bias of the judge. However, if bias increases expected effort the designer would still be able to impact the perception of bias by deciding when to overturn the judge.

Deciding how or whether to bias a contest toward a specific type of contestant is related to the policy of affirmative action. Chowdhury et al. (2020) gives a comprehensive overview of these policies and how results from the contest literature, some of which are mentioned above, inform the efficacy of the policies. This paper provides complimentary insights. The contest designer cannot choose bias and does not care about fairness of outcomes for their own sake. Instead, faced with a judge that is potentially biased, the designer wants to manage beliefs about the bias in order to increase contestant effort. We show that this goal and the goal of affirmative action are only sometimes aligned. In particular, the effort maximizing intervention can reduce the probability that a disadvantaged contestant wins.

Contests have been shown to be valuable when the judge or evaluator has different preferences from the designer but does not discriminate between types of decisions or agents. Frankel (2014) shows that ranking mechanisms are max-min optimal when the bias of the judge is unknown. Our approach is closer to that of Letina et al. (2020), which shows that a contest is optimal for judges who are known to be lenient, in that the designer's goal is to affect the actions of a third parties (the contestants) rather than the judge alone. However, rather than focusing on a bias which treats all contestants symmetrically, this paper focuses on a judge which favors certain *types* of agents.

Previous research has studied the impact of feedback on effort choice in dynamic contests. When the feedback gives information about relative position in the contest, a motivation effect may increase output prior to feedback and a discouragement effect may lower effort after (Aoyagi, 2010; Ederer, 2010). Private feedback about relative position can dominate public information when uncertainty is high (Mihm and Schlapp, 2019). When feedback yields information about contestant abilities,

[1]In dynamic contests, the bias imposed on one round may impact the effort choice in the others (Meyer, 1992; Ridlon and Shin, 2013; Barbieri and Serena, 2020). Including bias in a later period may reduce effort in that period but increase effort in the prior round.

sandbagging can lead to less effort from higher types prior to feedback, lowering expected effort (Zhang and Wang, 2009; Kubitz, 2022). In this paper, contestants are short-lived and therefore are only motivated by current beliefs, not by how current actions may affect future beliefs. This allows us to study how the release of interim information affects subsequent behavior without introducing signalling concerns into contestants' strategies. Our novel type of intervention by the designer seeks to reduce the feedback which discourages contestants.

More generally, the paper is related to the literature of information design where the contest designer can commit to a information disclosure rule over potential asymmetries between the contestants.[2] A majority of work focuses on optimal information disclosure about contestants' abilities or the number of contestants prior to a single contest (Lim and Matros, 2009; Fu et al., 2011; Feng and Lu, 2016; Zhang and Zhou, 2016; Zheng et al., 2018, 2019; Chen, 2020; Ryvkin and Drugov, 2020). Lu et al. (2018) and Serena (2019) identify optimal disclosure when only full or no disclosure is available. Our setting captures the common situation where future contestants can only observe past contest winners and the identity of those winners cannot be hidden by the contest designer. In this setting, the designer can only impact the information that future contestants have by changing the selection process and the identity of the winners of past contests.

The paper proceeds as follows. Section 2 describes the economic setting. Section 3 characterizes the solutions of both the static and dynamic versions of the model. Section 4 extends the analysis to the case where the designer prefers that subjects with higher ability win, then discusses assumptions of the model.

2 Model

A designer holds a contest for two short lived contestants each period for 2 periods. In each period, one contestant of type A and one of type B participates. The type of each contestant is publicly observable. During the period t in which they participate, each contestant can choose $a_{i,t} \in \{0, 1\}$ for that period. If they choose $a_{i,t} = 1$ (which we refer to as "putting forth effort"), they pay a cost c, and otherwise pay no cost. The (predetermined) prize in each period is v. Without loss of generality, we will assume that $c = 1$, since all contestants' incentives are only related to the ratio between v and c rather than the levels. We present the optimal contest in a single period as a benchmark and then

[2] See Bergemann and Morris (2019) and Kamenica (2019) for recent overviews of the information design literature.

compare it to the optimal dynamic contest.

Effort levels are observed by a judge who ranks the contestants and sends the ranking privately to the designer. The same judge evaluates participants in each contest. This judge may evaluate the contestants' output with bias. We say that the judge either *favors type A*, *favors type B*, or is *unbiased.* These characteristics are mutually exclusive and both the designer and all contestants have a common prior for these events of $p_{A,1}$, $p_{B,1}$, and $1 - p_{A,1} - p_{B,1}$ respectively.

The likelihood that the judge ranks a particular contestant first in any period depends on both contestants' efforts and the (potential) bias of the judge. In particular, if both contestants put forth effort then a judge biased toward type i ranks that type higher with probability one, and an unbiased judge reports each ranking with equal probability. If only one contestant puts forth effort the judge ranks this contestant first with probability one. Finally, if neither type puts forth effort, the judge reports each ranking with equal probability.[3]

Two comments about the judge's ranking process are in order. First, the potential bias of the judge and the effort of the contestant are complementary. Bias could have instead been modeled as a substitute for effort, where ties are broken for the favored type of contestant in the case where either both or neither contestant puts forth effort.[4] Second, the judge does not have direct preferences over the *outcome* of the contest but instead may have a preference to reward the effort of a particular type. These two properties of the ranking system are chosen to capture a judge that either prefers the effort of one type of contestant over the other or simply perceives that effort to be more valuable.[5] Importantly, the contest designer does not share this preference or perception.

The designer has a discount factor of $\delta < 1$ and maximizes the discounted sum of effort provided by the contestants.[6] Prior to the start of the first round, she can set the likelihood that each contestant

[3]A more general model would have the judge observing the following score in each period for an agent of type i:

$$b_i a_{i,t} + \varepsilon_{i,t},$$

where b_i is an unchanging bias parameter, $\varepsilon_{i,t}$ is a noisy shock, and the judge reports the contestant with the highest score to the designer. The reported model is then a limiting case of this structure, where $b_A > b_B$ with probability $p_{A,1}$, $b_B > b_A$ with probability $p_{B,1}$, and the variance of $\varepsilon_{i,t}$ goes to 0.

[4]While equilibria would be different and may involve mixed strategies in this model, we expect that many of the insights of our results will still be applicable: The designer still trades off between stronger incentives in the present and better beliefs in the future.

[5]For instance, if submissions are judged by a computer program or algorithm that produces rankings, the designer need not worry about strategic interference by the judge. Alternatively, the designer might also act as a judge knowing that they might display subconscious bias, but not know what or how strong that bias is.

[6]Restricting δ to be strictly less than one has the effect of eliminating equilibria in which the designer commits to overturning the judge enough in the first period such that only one type of contestant puts forth effort in that period. Doing so prevents any beliefs updates and ensures that both types put forth effort in the

receives the prize conditional on the judge's report. Let $\gamma_{i,t}$ be the probability that the contestant of type i wins the prize in period t given the judge ranks them first. This choice is public and the designer has the power of full commitment.[7]

In what follows, we focus on the Perfect Bayesian Equilibria (PBE) of the game between the designer and all contestants. A strategy of the designer is $\Gamma = (\gamma_{A,1}, \gamma_{B,1})$ in the one period game and $\Gamma = (\gamma_{A,1}, \gamma_{B,1}, \gamma_{A,2}, \gamma_{B,2})$ in the two period game. Strategies of each contestant consist of an action choice in the round each is active for each history, $a_{i,t}(h_t)$. Histories in each period are: $h_1 = \Gamma$ and $h_2 \in \{(\Gamma, A), (\Gamma, B)\}$ where the second argument indicates which type won the first contest. Beliefs about the judge's bias for each history are denoted $p_{i,t}(h_t)$. Note that $p_{i,1}(\Gamma) = p_{i,1}$ for all Γ.

For a given prior belief about the judge's bias, $(p_{A,1}, p_{B,1})$, a PBE consists of strategies for the designer and each contestant such that

1. Given the strategy of the designer, Γ, the contest outcomes, and conjectures about the actions of contestants, $\hat{a}_{i,t}$, beliefs about the judge's bias are updated using Bayes rule where possible.[8]

2. Actions, $a_{i,t}$, maximize the payoff of each contestant, $i = A, B$, in each period, $t = 1, 2$, given history, h_t, beliefs about the judges bias, $p_{i,t}(h_t)$, and conjectures of the action of the opposing contestant $\hat{a}_{-i,t}$.

3. The designer's strategy, Γ, maximizes the discounted expected effort of the contestants, $\mathbb{E}[a_{A,1} + a_{B,1} + \delta(a_{A,2} + a_{B,2})]$, given 1 and 2.

4. Actions equal conjectures, $a_{i,t} = \hat{a}_{i,t}$.

In cases in which one or more of the contestants is indifferent about whether to put forth effort, we assume that the contestant breaks that indifference in favor of putting forth effort.[9] We will also assume that $v > 2$ to focus on situations in which it's possible for both contestants to put forth effort in equilibrium. Due to the symmetric nature of the problem, our analysis in Section 3 is done under the

second period. The value of δ has no other effect on the equilibria presented in Section 3.

[7]We discuss the implications of commitment and what equilibria survive without it in Section 4.

[8]The only players who could cause a probability zero event to happen are the contestants in period 1. Since their payoffs do not depend on beliefs in the future, they will never have an incentive to change actions. Therefore it never matters what beliefs are after the zero probability event.

[9]This assumption has two implications. First, it eliminates cases of multiple equilibria over the measure-zero set of priors for which the designer's optimal choices must involve indifferent contestants. Second, and more substantively, it rules out equilibria in which contestants use mixed strategies. Such equilibria *can* be optimal for the designer in the two period model, but we consider the precise incentives required to induce such an equilibrium to be impractical.

assumption that the judge is more likely to be biased toward contestants of type A, i.e. $p_{A,1} \geq p_{B,1}$. Results for when $p_{B,1} > p_{A,1}$ directly follow.

3 Results

3.1 One Period Solution

Before presenting the solution to the one period problem, we define the *discouragement* due to bias as

$$D(v, p_{A,1}, p_{B,1}) = \frac{2}{v} \cdot \frac{1}{1 - (p_{A,1} - p_{B,1})}.$$

Discouragement plays an important role in the solutions of both the one period and two period models. Simply put, $D(v, p_{A,1}, p_{B,1})$ captures the discouraging effect that the judge's potential bias has on the less favored type, in this case type B. Discouragement increases with the difference in the likelihood that the two types are favored. However, this discouraging effect is weaker when the prize v is higher: even if a contestant thinks they are unlikely to win, they will still put forth effort if the prize is high enough.

With this definition of discouragement, describing the equilibrium of the single-period model is straightforward.

Theorem 1 *In any equilibrium of the single period model, both types of contestant put forth effort if*

$$D(v, p_{A,1}, p_{B,1}) \leq 1.$$

Otherwise, only type A puts forth effort. The designer sets $\gamma_{A,1}$ and $\gamma_{B,1}$ high enough so that neither contestant has the incentive to stop putting forth effort.

A graphical representation of the equilibrium described in Theorem 1 can be found in Figure 1. When one contestant is very likely to be favored while the other is very unlikely to be favored, only the contestant who is more likely to be favored puts forth effort. Essentially, the disparate beliefs have a discouraging effect on the contestant with a lower prior; he knows that even if he puts forth effort, he's unlikely to win. Discouragement happens for a wider range of initial beliefs when the prize is smaller. On the other hand, when $p_{A,1}$ and $p_{B,1}$ are similar and the other contestant is putting forth effort, switching from $a_{i,1} = 0$ to $a_{i,1} = 1$ increases the perceived likelihood of winning by roughly 50%, so both contestants put forth effort.

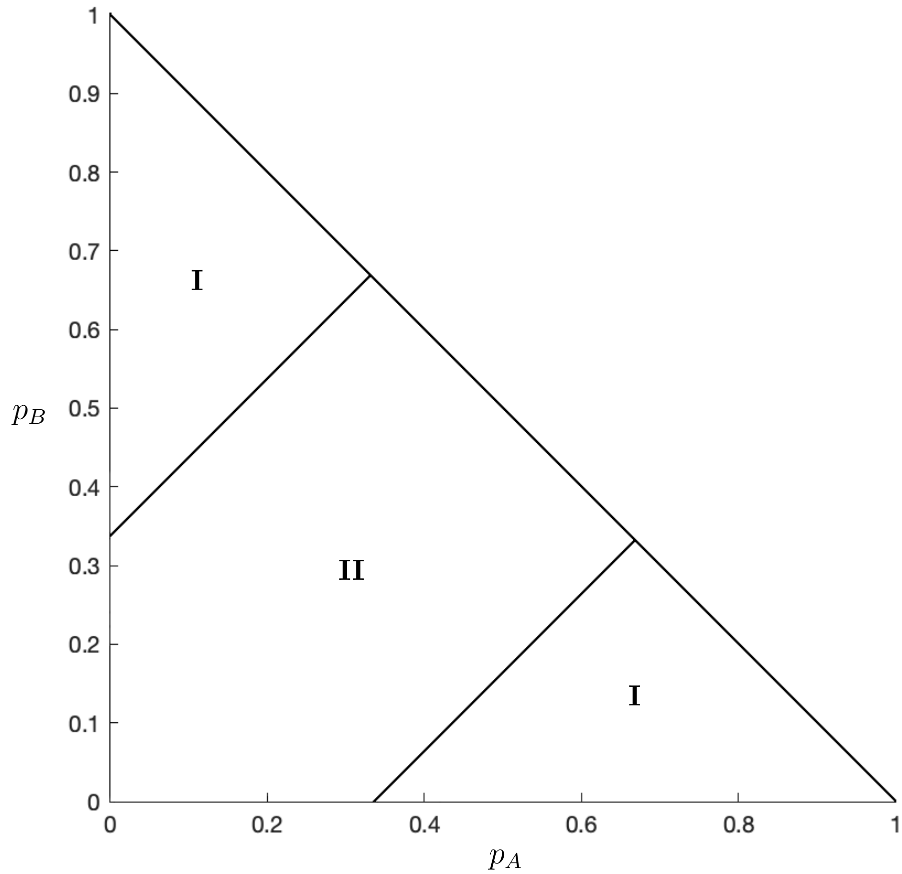


Figure 1: Solution to the single period model when $v = 3$. Both types of contestant put forth effort in the region labeled II, while only one type puts forth effort in the region labeled I.

One notable feature of the equilibrium described in Theorem 1 is that the designer has no use for setting $\gamma_{A,1}$ or $\gamma_{B,1}$ less than one. While the designer may be able to manipulate contestants' beliefs for the future by not giving the prize to the contestant the judge chose, doing so comes at a cost of weaker incentives in the current period. Because there is no continuation game in the single-period model, lowering $\gamma_{A,1}$ and $\gamma_{B,1}$ can only hurt the designer and reduce incentives to put forth effort.

3.2 Two Period Solution

In the static model, the designer had no reason to give the prize to the contestant which was not reported as the winner by the judge. Essentially, giving the prize to someone other than the reported winner simply weakens incentives without any additional benefits. Making the model dynamic introduces a new feature which may make it beneficial to give the prize to a different agent: reallocating the prize away from the reported winner can affect the beliefs of contestants in a later period, potentially causing them to put forth effort when they otherwise would not. Additionally, if one type of contestant winning the first contest leads to higher effort in the second contest, the designer can increase the probability of that type of contestant winning.

Figure 2 shows how beliefs update between the first and second period for the case of $\gamma_{1,A} = \gamma_{1,B} = 1$ when a contestant of type B wins in the first period. The starting point of the arrow indicates the prior beliefs, while the end of the arrow shows where beliefs are at the beginning of the second period. These posterior beliefs are on the vertical axis because there is zero possibility that the judge favors type A: given that both types put forth effort in the first period, observing type B win means that the judge either favors type B or is unbiased.

In the case of the blue arrow in Figure 2, beliefs updating does not cause a problem; even after the contestant of type B wins in the second period, both second period contestants think it is relatively likely that the judge is unbiased and are still willing to put forth effort. However, for both red arrows this updating process leads to pessimistic beliefs about contestants of Type A. In these cases, contestants of Type A would be unwilling to put forth effort in the second period.

More generally, from Bayes' rule, updated beliefs when both contestants are conjectured to exert effort in the first round must satisfy

$$\begin{aligned} p_{A,2}(\Gamma, A) &= \frac{\gamma_{A,1} p_{A,1}}{\frac{1}{2}\gamma_{A,1}(1 + p_{A,1} - p_{B,1}) + \frac{1}{2}(1 - \gamma_{B,1})(1 + p_{B,1} - p_{A,1})} \\ p_{B,2}(\Gamma, A) &= \frac{(1 - \gamma_{B,1}) p_{B,1}}{\frac{1}{2}\gamma_{A,1}(1 + p_{A,1} - p_{B,1}) + \frac{1}{2}(1 - \gamma_{B,1})(1 + p_{B,1} - p_{A,1})}. \end{aligned} \tag{1}$$

Setting $\gamma_{A,1}$ or $\gamma_{B,1}$ to less than one introduces noise into the learning process and causes contestants to update less. Specifically, the posterior belief about bias toward the losing type of contestant is not as low and toward the winning type of contestant is not as high. For $\gamma_{A,1} = \gamma_{B,1} = \frac{1}{2}$, the prize is assigned in a way which is uncorrelated with the judge's report and beliefs do not update at all.

Lemma 1 *The following relationships between posterior beliefs about the judges bias and the designer's strategy hold when both contestants are conjectured to exert effort in the first contest and* $\gamma_{i,1} > 1/2$

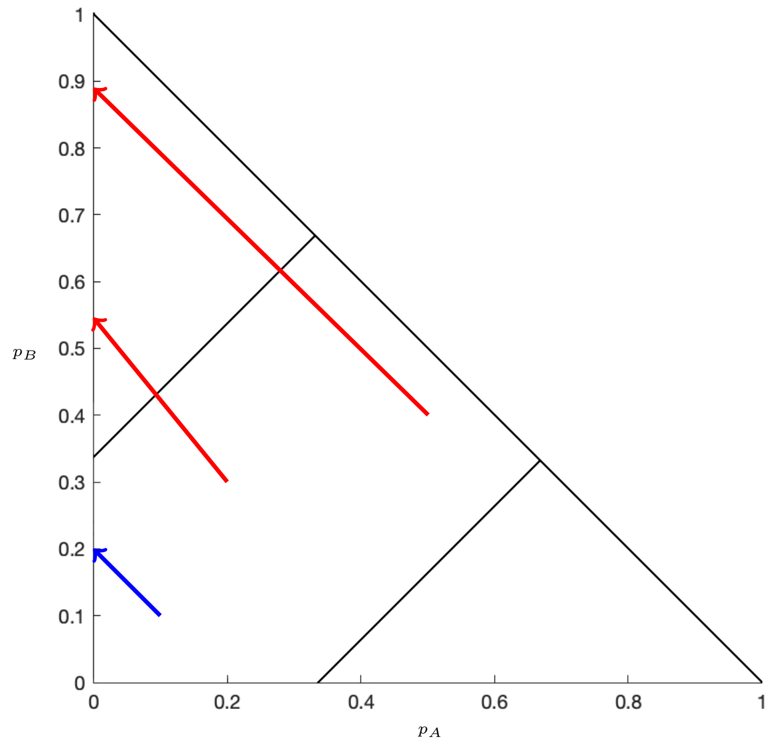


Figure 2: Updated beliefs after the type B contestant wins in the first period without intervention from the designer. For the blue arrow, both types of contestant put forth effort in the second period. For the red arrows, only contestants of type B put forth effort in the second period.

for $i = A, B$.

$$\frac{\partial p_{i,2}(\Gamma, i)}{\partial \gamma_{-i,1}} > \frac{\partial p_{i,2}(\Gamma, i)}{\partial \gamma_{i,1}} > 0 \text{ and } \frac{\partial p_{i,2}(\Gamma, -i)}{\partial \gamma_{i,1}} < \frac{\partial p_{i,2}(\Gamma, -i)}{\partial \gamma_{-i,1}} < 0.$$

Given that the contestant of type B is putting forth effort in period t, a contestant of type A only puts forth effort if

$$\gamma_{A,t} + \gamma_{B,t} - 1 \geq \frac{2}{v} \cdot \frac{1}{1 + p_{A,t}(h_t) - p_{B,t}(h_t)}. \tag{2}$$

This captures the features which are payoff-relevant to a contestant in a given period. For a contestant of type i, higher values of $p_{i,t}(h_t)$, $\gamma_{i,t}$, and $\gamma_{j,t}$ make it more valuable to put forth effort, while $p_{i,t}(h_t)$ makes effort less valuable.

The cost of lowering $\gamma_{A,t}$ or $\gamma_{B,t}$ is weaker incentives in period t, while the benefit arises from "better" beliefs (from the point of view of the designer) in later periods. Thus, in the final period the designer does not benefit from lowering either value, and sets both equal to one.

Lemma 2 *It is optimal for the designer to set* $\gamma_{A,2} = \gamma_{B,2} = 1$ *in the second period.*

Beliefs about whom the judge favors are updated after observing whom the prize is given to when both types are putting forth effort in the first period. However, this is not true when at least one type doesn't put forth effort. In this case, beliefs do not update for either type of contestant no matter whom the prize is given to. Given that only one contestant put forth effort, the judge reports that they are the winner with probability one, and the only way that the other contestant could be awarded the prize is if the designer went against the judge's recommendation. Combining this with the fact that $\delta < 1$, it is never optimal for the designer to decrease $\gamma_{A,1}$ or $\gamma_{B,1}$ to increase effort in future periods at the cost of lowering effort today. Thus, for all equilibria where prior beliefs are in region II, the designer's strategy must be such that both contestants exert effort in the first period. We can combine the inequalities given in (2) in period one for both types to get this condition which characterizes the minimum levels of $\gamma_{A,1}$ and $\gamma_{B,1}$ which ensure that both constants put forth effort in the first period.

$$D(v, p_{A,1}, p_{B,1}) \leq \gamma_{A,1} + \gamma_{B,1} - 1 \tag{IC-1}$$

While beliefs about who is favored are updated when the first period's prize is awarded, whether those posteriors still allow for effort provision in the second period depends on both initial beliefs and the designer's strategy, Γ. We combine the updating rules (1) with the incentive constraints given in inequality (2) to find

$$\gamma_{A,1} - \frac{\left(1 - \frac{2}{v}\right)(1 + p_{B,1} - p_{A,1}) - 2p_{B,1}}{\left(1 - \frac{2}{v}\right)(1 + p_{A,1} - p_{B,1}) + 2p_{A,1}} \gamma_{B,1} \leq 1 \tag{BC-A}$$

$$\gamma_{B,1} - \frac{\left(1 - \frac{2}{v}\right)(1 + p_{A,1} - p_{B,1}) - 2p_{A,1}}{\left(1 - \frac{2}{v}\right)(1 + p_{B,1} - p_{A,1}) + 2p_{B,1}} \gamma_{A,1} \leq 1. \tag{BC-B}$$

These *belief constraints*, (BC-A) and (BC-B), characterize the values of $\gamma_{A,1}$ and $\gamma_{B,1}$ for which A and B (respectively) put forth effort in the second period conditional on their type losing. Specifically, they ensure that the beliefs about type i do not fall too much upon observing type i lose. In principle we must also check that beliefs about type i do not fall too much upon observing type i win. However, one can show that beliefs only fall in this way if $\gamma_{A,1} + \gamma_{B,1} < 1$. Such values of $\gamma_{A,1}$ and $\gamma_{B,1}$ would violate (IC-1), and thus will never hold in equilibrium.

We interpret the coefficient on $\gamma_{B,1}$ in (BC-A) and on $\gamma_{A,1}$ in (BC-B) as the *confidence* that types A and B respectively have that the judge does not favor the other type. It will be convenient to denote this as

$$C_A(v, p_{A,1}, p_{B,1}) = \frac{\left(1 - \frac{2}{v}\right)(1 + p_{B,1} - p_{A,1}) - 2p_{B,1}}{\left(1 - \frac{2}{v}\right)(1 + p_{A,1} - p_{B,1}) + 2p_{A,1}}$$

and

$$C_B(v, p_{A,1}, p_{B,1}) = \frac{\left(1 - \frac{2}{v}\right)(1 + p_{A,1} - p_{B,1}) - 2p_{A,1}}{\left(1 - \frac{2}{v}\right)(1 + p_{B,1} - p_{A,1}) + 2p_{B,1}}.$$

We say that type i is *confident* if $C_i(v, p_{A,1}, p_{B,1}) \geq 0$. This cutoff is relevant because when it is satisfied, a contestant of type i is willing to put forth effort in the second round after observing their type lose *even when* $\gamma_{A,1} = \gamma_{B,1} = 1$.[10] Furthermore, for more negative values of $C_A(v, p_{A,1}, p_{B,1})$ and $C_B(v, p_{A,1}, p_{B,1})$, the set of values of $\gamma_{A,1}$ and $\gamma_{B,1}$ which satisfy (BC-A) and (BC-B) is smaller.

The incentive constraint for the first period (IC-1) and belief constraints for the second period (BC-A) and (BC-B) capture the designer's problem. Lowering $\gamma_{A,1}$ or $\gamma_{B,1}$ may help to satisfy (BC-A) or (BC-B). However, doing so cannot come at the cost of violating (IC-1). Thus, the solution to the designer's problem will depend on the priors, which determine whether (BC-A) and/or (BC-B) can be satisfied without violating (IC-1).

Our first result characterizes the equilibrium of the two period game when neither type A nor B is confident. In this case, without intervention from the designer, the contestant whose type lost in the first period would never put forth effort in the second period. We suppress the arguments of the discouragement and confidence functions for the remainder of the section for expositional ease.

Theorem 2 *Suppose that no type of contestant is confident.*

- *If* $D \leq \frac{(1+C_B)(1+C_A)}{1-C_A C_B}$*, then in any PBE both types of contestant put forth effort in both periods and the designer occasionally overturns the judge.*
- *If* $\frac{(1+C_B)(1+C_A)}{1-C_A C_B} < D \leq 1 + C_B$*, then in any PBE only contestants of type B put forth effort in the second period conditional on their type losing in the first. The designer sets* $\gamma_{A,1} = 1$*, and*

[10] This can be seen by considering (BC-A). When $C_A(v, p_{A,1}, p_{B,1}) \geq 0$, the inequality holds for *any* values of $\gamma_{A,1}$ and $\gamma_{B,1}$ that are between 0 and 1.

$\gamma_{B,1}$ to the minimum level possible so that both types put forth effort in the first period.

- *If $1 + C_B < D \leq 1 + C_A$, then in any PBE only contestants of type A put forth effort in the second period conditional on their type losing in the first. The designer sets $\gamma_{B,1} = 1$, and $\gamma_{A,1}$ to the minimum level possible so that both types put forth effort in the first period.*

- *If $1 + C_A < D \leq 1$, then in any PBE neither type of contestant puts forth effort in the second period conditional on their type losing in the first. The designer sets $\gamma_{A,1}$ and $\gamma_{B,1}$ high enough so that neither contestant has the incentive to stop putting forth effort in the first period.*

- *If $1 < D$, then in any PBE contestants of type A put forth effort in both periods but contestants of type B do not put forth effort in either period.*

The way in which equilibria vary with primitives in Theorem 2 is intuitive. When discouragement is low and contestants' confidence is not too low, the designer is able to induce both types of contestant to put forth effort in both periods by occasionally overturning the judge. For higher levels of discouragement or lower levels of confidence, the designer is able to extract correspondingly less effort from the contestants.

The sets of parameters for associated with the types of equilibria described in Theorem 2 can be found in Figure 3. The non-grey region corresponds to parameter values such that neither type is confident and $p_{A,1} \geq p_{B,1}$. Each color represents a different form of the equilibria from Theorem 2: $D \leq \frac{(1+C_B)(1+C_A)}{1-C_A C_B}$ is in light blue, $\frac{(1+C_B)(1+C_A)}{1-C_A C_B} < D \leq 1 + C_B$ is in green, $1 + C_B < D \leq 1 + C_A$ is in yellow, $1 + C_A < D \leq 1$ is in orange, and $1 < D$ is in red. Note that when $C_A < 0$, then $p_A \geq p_B$ implies that $C_A \geq C_B$, so Theorem 2 characterizes all of these cases.

Figure 4 graphically shows how the three constraints determine what values of $\gamma_{A,1}$ and $\gamma_{B,1}$ should be chosen for fixed values of $p_{A,1}$, $p_{B,1}$, and v. The graph in panel (a) corresponds to the case in which neither type of contestant is confident but $D \leq \frac{(1+C_B)(1+C_A)}{1-C_A C_B}$. The shaded blue triangle is the set of γ which satisfies (IC-1), (BC-A), and (BC-B). Thus, *any* values of $\gamma_{A,1}$ and $\gamma_{B,1}$ which lie within this triangle guarantee that both types of contestant put forth effort in both periods.

One portion of Theorem 2 which merits further discussion is the range of priors for which $\frac{(1+C_B)(1+C_A)}{1-C_A C_B} < D \leq 1 + C_B$. The fact that $\frac{(1+C_B)(1+C_A)}{1-C_A C_B} < D$ indicates that the designer cannot simultaneously incentivize *both* types to put forth effort conditional on losing, but since $D \leq 1 + C_B$ and $D \leq 1 + C_A$, the designer can incentivize *either*. This can be seen in panel (b) of Figure 4, where values of $\gamma_{A,1}$ and $\gamma_{B,1}$ in the orange region guarantee that the contestant of type A puts forth effort

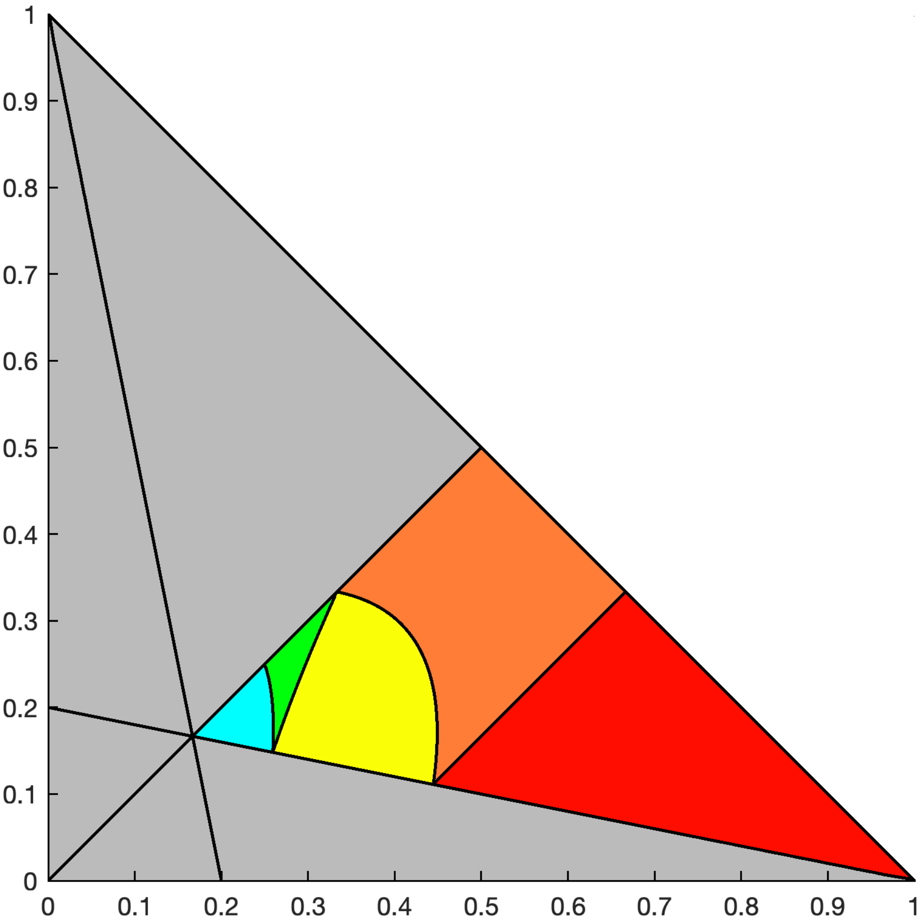
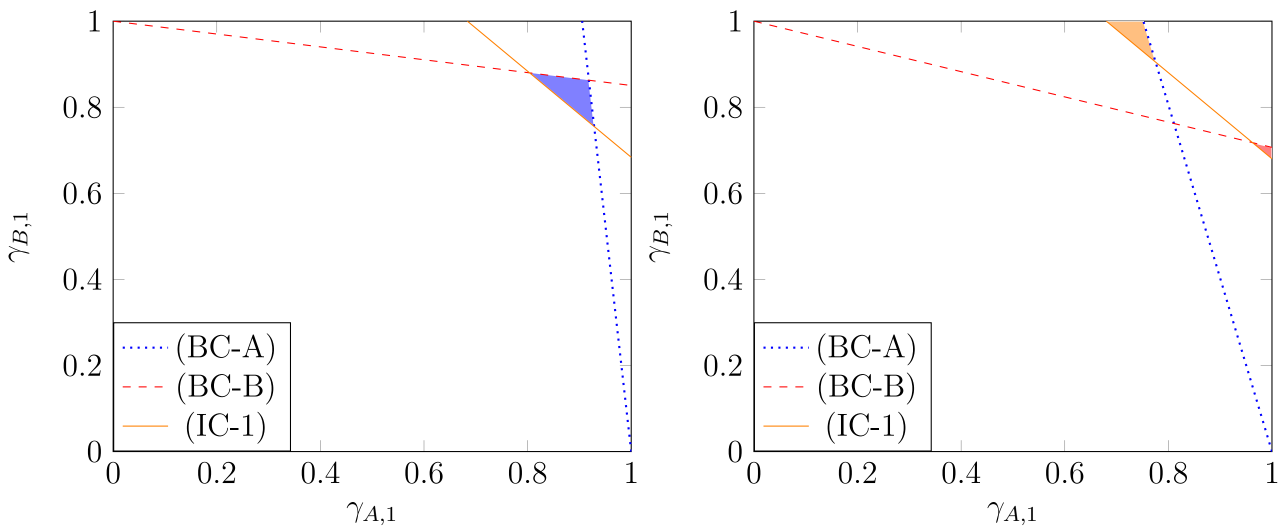


Figure 3: The colorful regions of this figure show how the form of the equilibrium varies with priors when $v = 3$ and neither type is confident. Total effort provision decreases from left to right. The designer overturns the judge in the light blue, green, and yellow regions.

conditional on losing in the first period, while this is true for type B in the red region. Since only one type will put forth effort conditional on losing, the designer prefers to choose the type which can be induced to lose the most often, because this increases the likelihood both types exert effort in the second period. To do this, the designer's intervention ensures that the type which is disfavored *a priori* puts in effort in the second period even after that type loses the first period. Furthermore, the optimal way to do this is to minimize the chance that this type wins conditional on both parties still having an incentive to put forth effort in the first period (in this case, setting $\gamma_{B,1}$ to the minimum value which does not violate (IC-1)). The optimal choice of $\gamma_{A,1}$ and $\gamma_{B,1}$ in this case is exactly the point where $\gamma_{A,1} = 1$ on the line representing (IC-1).

Reducing $\gamma_{B,1}$ while keeping $\gamma_{A,1} = 1$ has two impacts that benefit the designer. The first is the direct impact of having contestant B lose more often. As type A will not put in effort in the second



(a) $p_{A,1} = 0.225$, $p_{B,1} = 0.2$, $v = 3$

(b) $p_{A,1} = 0.3$, $p_{B,1} = 0.28$, $v = 3$

Figure 4: This figure shows the interaction of three constraints for two sets of parameters. In panel (a), the blue triangle represents the values of $(\gamma_{A,1}, \gamma_{B,1})$ which satisfy (BC-A), (BC-B), and (IC-1). In panel (b), values within the orange triangle satisfy (BC-A) and (IC-1), while values in the red triangle satisfy (BC-B) and (IC-1). No values in panel (b) satisfy all three constraints.

period after a loss, the designer wishes to minimize the probability that this happens. Secondly, the best way to make type B more optimistic about the second period after losing the first period is to reduce $\gamma_{B,1}$. This can be seen from Lemma 1. Decreasing $\gamma_{B,1}$ both increases $p_{B,2}(\Gamma, A)$ and decreases $p_{A,2}(\Gamma, A)$ at a faster rate than decreasing $\gamma_{A,1}$. Intuitively, a bad outcome for B can more easily attributed to (bad) luck for lower values of $\gamma_{B,1}$; a loss by type B is more likely to have been caused by the judge being overruled by the designer when B was actually ranked higher.

The next result characterizes the equilibrium of the two period game when type A is confident but type B is not.

Theorem 3 *Suppose that contestants of type A are confident but contestants of type B are not confident.*

- *If $D \leq 1 + C_B$, then in any PBE both types of contestant put forth effort in both periods and the designer occasionally overturns the judge.*
- *If $1 + C_B < D \leq 1$, then in any PBE both types of contestant put forth effort in the first period, but only contestants of type A put forth effort in the second period conditional on their type losing in the first. The designer sets $\gamma_{B,1} = 1$, and $\gamma_{A,1}$ to the minimum level possible so that*

both types put forth effort in the first period.

- *If* $1 < D$*, then in any PBE contestants of type* A *put forth effort in both periods but contestants of type* B *do not put forth effort in either period.*

Given that type A is confident, the designer does not need to intervene in order for contestants of type A put forth effort in both periods. Instead, the form of the equilibrium depends on the discouragement and confidence of type B. When confidence is not too low, only a small decrease in $\gamma_{B,1}$ is necessary for (BC-B) to be satisfied. This small decrease does not lead to a violation of (IC-1) when D is not too high. For lower confidence levels, the intervention necessary to incentivize second period effort from type B conditional on them losing in the first period becomes infeasible. Finally, if discouragement is too high, type B is not even willing to put forth effort in the first period.

A graphical depiction of the of the regions described in Theorem 3 can be found in Figure 6. The non-grey region corresponds to parameter values such that type A is confident, type B is not, and $p_{A,1} \geq p_{B,1}$. Each color represents a different form of the equilibria from Theorem 2: $D \leq 1 + C_B$ is in light blue, $1 + C_B < D \leq 1$ is in yellow, and $1 < D$ is in red.

Finally, we characterize the equilibrium of the two period game when *both* types are confident.

Theorem 4 *Suppose that both types of contestant are confident. Then both types put forth effort in both periods. The designer sets* $\gamma_{A,1}$ *and* $\gamma_{B,1}$ *high enough so that neither contestant has the incentive to stop putting forth effort in the first period.*

This result is straightforward. Since both types are confident, both types have the incentive to put forth effort in the second period conditional on their type losing in the first period. This is true even without interference from the designer. The only point left to check is that both types have the incentive to put forth effort in the first period, but (IC-1) is always satisfied with $\gamma_{A,1} = \gamma_{B,1} = 1$ when both types are confident. The region for which both types of contestant are confident and $p_{A,1} \geq p_{B,1}$ can be seen in Figure 7.

4 Extensions & Discussion

4.1 Extension: Screening

In the model presented in Section 2, the designer's objective was to maximize the effort that the contestants put forth. In addition to incentivizing effort, contests are also used to identify the "best"

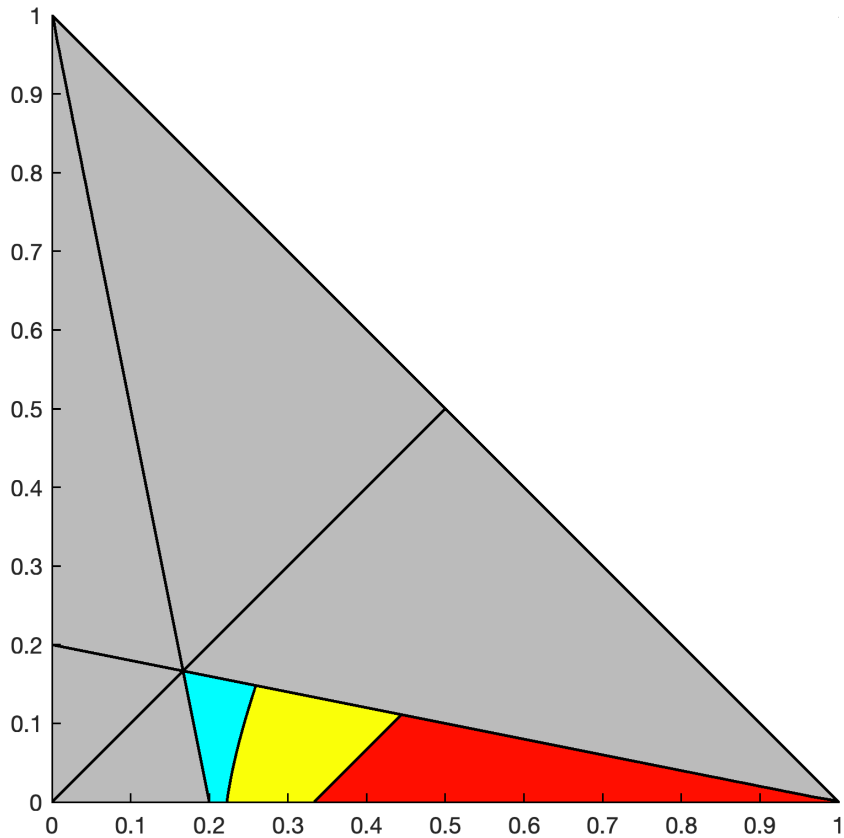


Figure 6: The colorful regions of this figure show how the form of the equilibrium varies with priors when $v = 3$, type A is confident, and type B is not. Total effort provision decreases from left to right. The designer overturns the judge in the light blue region.

contestant, i.e. to screen contestants' ability. Indeed, returning to the leading example we gave in Section 1, it is likely that in addition to using a contest to incentivize the interns to put forth effort, the firm would also prefer that the intern with higher ability wins the contest, as they are given the permanent position as the prize.

Thus, we extend our model to introduce ability on the part of contestants and extend the designer's objective function to account for the desire to assign the prize to high types. Specifically, we assume that contestants have high skill with probability $\mu \in (0, 1)$ and low skill with probability $1 - \mu$. We assume that skill levels are not directly observable to anyone in the game, including the contestants themselves.

While no one can directly observe the skill levels of the contestants, we assume that an unbiased

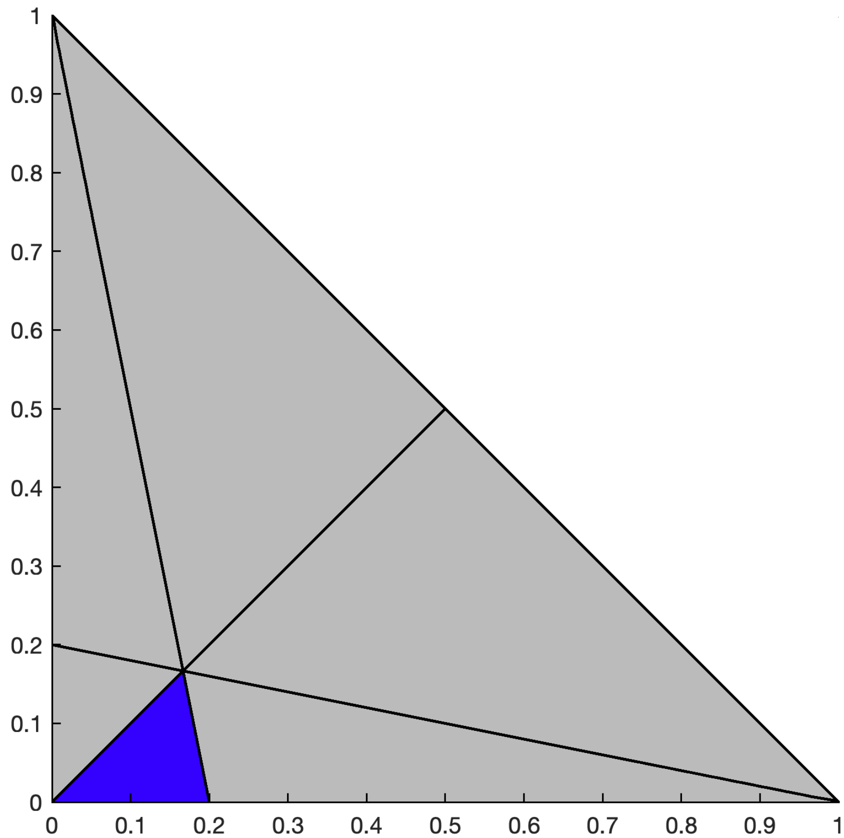


Figure 7: The blue region of this figure shows the priors for which both types are confident when $v = 3$.

judge is able to correctly identify whether one contestant has a higher skill level than the other when both contestants put forth effort. To be more precise, if both contestants put forth effort, one contestant has high skill, and the other contestant has low skill, an unbiased judge ranks the contestant with high skill first. If the judge favors type i and both contestants put forth effort, then the judge ranks type i first with probability one. If only one contestant puts forth effort any judge will rank that contestant first, and if neither contestant puts forth effort any judge will report each ranking with equal probability.[11]

In addition to valuing effort, as before, the designer prefers that the prize is awarded to a contestant with high skill. We introduce a single parameter $\lambda \in [0, 1]$ that captures the relative weight

[11]In this section, we still focus on the case in which $p_{A,1} > p_{B,1}$.

that the designer places on effort. Specifically, in each period t, the designer receives

$$\lambda(e_{A,t} + e_{B,t}) + (1 - \lambda)\mathbb{P}(\text{period } t \text{ winner is high type}).$$

Furthermore, we again study optimal contest design when the designer's instrument is the ability to commit to stochastically overturning the judge (i.e. setting $\gamma_{i,t} < 1$). This nests the model that was studied in Sections 2 and 3 with the case of $\lambda = 1$.

We first show how this extension affects the equilibrium in the single contest model.

Proposition 1 *In the equilibrium of the single period model with screening and $\lambda < 1$, both contestants put forth effort if*

$$D(v, p_{A,1}, p_{B,1}) \leq 1.$$

Otherwise, only type A puts forth effort. The designer sets $\gamma_{A,1}$ and $\gamma_{B,1}$ equal to 1 if both types are putting forth effort.

Proposition 1 is essentially identical to Theorem 1, except that the designer is guaranteed to respect the judge's ranking when both types are putting forth effort. The reason for this is intuitive. As before, lowering $\gamma_{i,t}$ weakens incentives to put forth effort in period t, and there is no subsequent improvement in beliefs if there is only one contest in the game. The difference from the model without skill is that when the designer cares about the skill level of the winner, overturning the judge comes at an additional cost - any judge that may be unbiased is more likely than not assigning the prize to the contestant with high skill, given that the contestants have differing skill levels and are both putting forth effort.

The intuition behind the result in Proposition 1 suggests a broader idea, namely that overturning the judge is more costly to the designer when there are concerns about the skill of the contest's winner. With this intuition, we can confirm that if the designer would not benefit from overturning the judge in the two period model *without* contestant skill, then the designer will never overturn the judge in the two period model *with* contestant skill.

Proposition 2 *In the equilibrium of the two period model with screening and $\lambda < 1$, the designer maximizes her payoffs by setting $\gamma_{i,t} = 1$ for all i and t if (1) neither type is confident and $1 + C_A < D$, (2) type A is confident and $1 < D$, or (3) both types are confident.*

The results stated in Propositions 1 and 2 show that, in a sense, incentives to overturn the judge in order to improve beliefs are weaker when there are screening concerns in addition to a desire to increase effort: If the designer would not overturn the judge without screening, then she does not do it with screening either. However, the core feature that makes overturning the judge valuable in this model—the fact that it can induce greater participation and effort by preventing contestants from getting discouraged—is still present. We show this in the next proposition.

Proposition 3 *Suppose that neither type is confident and $D \leq \frac{(1+C_B)(1+C_A)}{1-C_AC_B}$. Then if*

$$\delta \geq \frac{(1-\lambda)(1-\mu)\mu(1-p_{i,1}-p_{j,1})(2C_AC_B+C_A+C_B)}{[\lambda+(1-\lambda)(\mu^2+\mu(1-\mu)(2-p_{i,1}-p_{j,1})-\mu)](C_AC_B-1)},$$

committing to occasionally overturn the judge increases the designer's payoffs.

Proposition 3 states that for some priors about the judge's bias, the designer is made better off by committing to overturn the judge if she is patient enough. The intuition for this result follows from the above discussion. Overturning the judge induces more effort and an increased likelihood of a high type winning in the second contest, but at the cost of lowering the likelihood of a high type winning in the first contest.

Corollary 1 provides a simplification of Proposition 3 for the case in which the designer *only* cares about the skill level of the winning contestant and does not directly value contestants' effort.

Corollary 1 *Suppose that neither type is confident, $\lambda = 0$, and $D \leq \frac{(1+C_B)(1+C_A)}{1-C_AC_B}$. Then there exists a $\delta^* \in (0,1)$ such that if $\delta > \delta^*$, committing to occasionally overturn the judge increases the designer's payoffs.*

The intuition is straightforward. Without ever overturning the judge, neither type of contestant puts in effort in the second contest when the contestant of the other type wins the first contest, and the probability that a high type is selected in the second contest is equal to the prior. Because $D \leq \frac{(1+C_B)(1+C_A)}{1-C_AC_B}$ the designer can induce both types of contestant to put forth effort in the second period by only sometimes overturning the judge in the first period. Then in the case where the judge is unbiased and only the type that lost the first contest is high ability in the second contest, the designer would benefit from the judge identifying the higher ability contestant. If the designer values the two periods roughly equally (δ near one), committing to stochastically overturn the judge in the first contest improves the designer's expected payoffs.

4.2 Discussion

We study a dynamic contest in which sequentially arriving contestants learn about the contest judge's bias by observing who has won in the past. We allow a contest designer to manipulate who is awarded a prize in order to manage contestants' beliefs and maximize effort. We show that for some prior beliefs about the judge's bias, the designer can increase total expected effort. Specifically, there are beliefs for which this intervention can ensure that all contestants put forth effort conditional on losing when neither (light blue area in Figure 3) or only one (light blue area in Figure 6) would without it. For priors where intervention can only induce one type of contestant to exert effort in the contest, she prefers to choose the one which is more likely to face bias from the judge (green area in Figure 3). The optimal intervention in this case lowers this type of contestants likelihood of receiving the prize in the first round even further. Lastly, where no intervention can prevent the type more likely to face bias from not exerting effort after losing the first contest, we show the designer can benefit by reducing the likelihood this type loses the first round (yellow area in Figures 3 and 6). We also report an extension of the model in which the designer uses the contest to screen contestants' ability, and show that while overturning the judge becomes more costly, the incentive to overturn the judge remains in some cases.

The results from Theorem 2 demonstrate the difficulty caused by the signal space being restricted to the report of the winner. Regardless of the intervention chosen in our setting, there are at most two signals that contestants can observe after the first round. Manipulations of the judge's report impact the informativeness of each signal and how often each occurs. Interventions to maximize effort can harm the type of contestant more likely facing bias. In this case, the designer wishes to reduce the informativeness of the signal that is more likely to happen, namely that this type loses. This is done by artificially increasing the likelihood of this event - overturning rankings when this disadvantaged type is chosen by the judge.

In settings where a more continuous measure of quality could be sent (e.g. one in which a continuous "score" is generated and observed from each contestant's submission) this incentive on the part of the designer need not be present. More generally, the question of what signals should be revealed in various environments is likely to be a fruitful path for future research which draws upon both the dynamic contest and information design literatures. We show how the designer is able to manipulate information in this realistic setting where only basic outcomes of past contests are publicly available.

We conclude by discussing a few of the assumptions of the model.

The contest designer has the power to publicly commit at the beginning of the game to a strategy with randomization that depends on the judge's report. This ability to commit is important at several points. If the randomization is chosen *after* contestants choose their effort in the first period, the designer would sometimes benefit from ignoring (IC-1) and assigning the prize in a way unrelated to the judge's report. Furthermore, without the commitment to respect the outcome of the randomization, equilibria with intervention would only remain when total effort is the same in the second contest given either outcome from the first contest (light blue area of Figures 3 and Figure 6).

Our analysis also assumes the designer knows the precise common prior beliefs of the contestants. The assumption is easy to (partially) relax without qualitatively changing the results. Some values of $\gamma_{A,1}$ and $\gamma_{B,1}$ solve the designer's problem for a range of the contestants' prior beliefs. Thus, as long as the designer's beliefs about the contestants' priors fall within this range, the same choice of of $\gamma_{A,1}$ and $\gamma_{B,1}$ solve the designer's problem. Inducing additional uncertainty on the part of the designer is an interesting pathway for future research.

The restriction to a two period model is another restriction that brings with it meaningful consequences: the designer's problem is centered around mitigating the effects of a loss on one type of contestant. This leaves no space for intuitive strategies such as occasionally awarding the prize to the type which has lost several past contests in order to make them less pessimistic. We expect such strategies to be valuable to the designer when there are more than two successive contests.

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A Proofs

Theorem 1 *In any equilibrium of the single period model, both types of contestant put forth effort if*

$$D(v, p_{A,1}, p_{B,1}) \leq 1.$$

Otherwise, only the type more likely to be favored puts forth effort. The designer sets $\gamma_{A,1}$ and $\gamma_{B,1}$ high enough so that neither contestant has the incentive to stop putting forth effort.

Proof. If the contestant of type B is not putting forth effort, the type A contestant puts forth effort if

$$\mathbb{P}(A \text{ receives prize}|a_{A,1} = 1, a_{B,1} = 0)v - 1 \geq \mathbb{P}(A \text{ receives prize}|a_{A,1} = 0, a_{B,1} = 0)v.$$

We can substitute in the judge's decision rule to get

$$\gamma_{A,1}v - 1 \geq \left[\frac{1}{2}\gamma_{A,1} + \frac{1}{2}(1 - \gamma_{B,1})\right] v.$$

which simplifies to

$$\frac{2}{v} \leq \gamma_{A,1} + \gamma_{B,1} - 1.$$

Notice that this equality also holds for type B putting forth effort when A is not.

If the contestant of type B is putting forth effort, the type A contestant puts forth effort if

$$\mathbb{P}(A \text{ receives prize}|a_{A,1} = 1, a_{B,1} = 1)v - 1 \geq \mathbb{P}(A \text{ receives prize}|a_{A,1} = 0, a_{B,1} = 1)v.$$

We can substitute in the judge's decision rule to get

$$\left[p_{A,1}\gamma_{A,1} + p_{B,1}(1 - \gamma_{B,1}) + \frac{1}{2}(1 - p_{A,1} - p_{B,1})(1 + \gamma_{A,1} - \gamma_{B,1})\right] v - 1 \geq (1 - \gamma_{B,1})v$$

which simplifies to

$$\frac{2}{v}\left(\frac{1}{1 + p_{A,1} - p_{B,1}}\right) \leq \gamma_{A,1} + \gamma_{B,1} - 1$$

with a symmetric inequality holding for the contestant of type B.

Thus, increasing $\gamma_{A,1}$ and $\gamma_{B,1}$ increases the incentive to put forth effort for both contestants in all situations, so without loss of generality we can assume that the designer will set $\gamma_{A,1} = \gamma_{B,1} = 1$. Since $v > 2$, this shows that there is no equilibrium in which neither contestant puts forth effort and the inequality given in the theorem immediately follows. ■

Lemma 1 *The following relationships between posterior beliefs about the judges bias and the designer's strategy hold when both contestants are conjectured to exert effort in the first contest and* $\gamma_{i,1} > 1/2$ *for* $i = A, B$.

$$\frac{\partial p_{i,2}(\Gamma, i)}{\partial \gamma_{-i,1}} > \frac{\partial p_{i,2}(\Gamma, i)}{\partial \gamma_{i,1}} > 0 \text{ and } \frac{\partial p_{i,2}(\Gamma, -i)}{\partial \gamma_{i,1}} < \frac{\partial p_{i,2}(\Gamma, -i)}{\partial \gamma_{-i,1}} < 0.$$

Proof. The posteriors when both contestants are conjectured to put in effort in period one are

$$p_{i,2}(\Gamma, i) = \frac{\gamma_{i,1} p_{i,1}}{\frac{1}{2}\gamma_{i,1}(1 + p_{i,1} - p_{-i,1}) + \frac{1}{2}(1 - \gamma_{-i,1})(1 + p_{-i,1} - p_{i,1})}$$
$$p_{i,2}(\Gamma, -i) = \frac{(1 - \gamma_{i,1}) p_{i,1}}{\frac{1}{2}\gamma_{-i,1}(1 + p_{-i,1} - p_{i,1}) + \frac{1}{2}(1 - \gamma_{i,1})(1 + p_{i,1} - p_{-i,1})}.$$

Taking partial derivatives

$$\frac{\partial p_{i,2}(\Gamma, i)}{\partial \gamma_{i,1}} = \frac{\frac{1}{2}(1 - \gamma_{-i,1})(1 + p_{-i,1} - p_{i,1})}{\left(\frac{1}{2}\gamma_{i,1}(1 + p_{i,1} - p_{-i,1}) + \frac{1}{2}(1 - \gamma_{-i,1})(1 + p_{-i,1} - p_{i,1})\right)^2}$$
$$\frac{\partial p_{i,2}(\Gamma, i)}{\partial \gamma_{-i,1}} = \frac{\frac{1}{2}\gamma_{i,1}(1 + p_{-i,1} - p_{i,1})}{\left(\frac{1}{2}\gamma_{i,1}(1 + p_{i,1} - p_{-i,1}) + \frac{1}{2}(1 - \gamma_{-i,1})(1 + p_{-i,1} - p_{i,1})\right)^2}$$
$$\frac{\partial p_{i,2}(\Gamma, -i)}{\partial \gamma_{i,1}} = \frac{-\frac{1}{2}\gamma_{-i,1}(1 + p_{-i,1} - p_{i,1})}{\left(\frac{1}{2}\gamma_{-i,1}(1 + p_{-i,1} - p_{i,1}) + \frac{1}{2}(1 - \gamma_{i,1})(1 + p_{i,1} - p_{-i,1})\right)^2}$$
$$\frac{\partial p_{i,2}(\Gamma, -i)}{\partial \gamma_{-i,1}} = \frac{-\frac{1}{2}(1 - \gamma_{i,1})(1 + p_{-i,1} - p_{i,1})}{\left(\frac{1}{2}\gamma_{-i,1}(1 + p_{-i,1} - p_{i,1}) + \frac{1}{2}(1 - \gamma_{i,1})(1 + p_{i,1} - p_{-i,1})\right)^2}$$

The result follows from $\gamma_{i,1} > 1/2$ and $\gamma_{-i,t} > 1/2$. ■

Lemma 2 *It is optimal for the designer to set* $\gamma_{A,2} = \gamma_{B,2} = 1$ *in the second period.*

Proof. Notice that in the second period, the analysis of whether each contestant will put forth effort is exactly equivalent to the analysis from the single period game in the proof of Theorem 1, except that $p_{A,1}$, $p_{B,1}$, $\gamma_{A,1}$, and $\gamma_{B,1}$ are replaced by $p_{A,2}$, $p_{B,2}$, $\gamma_{A,2}$, and $\gamma_{B,2}$. Thus increasing $\gamma_{A,2}$ and $\gamma_{B,2}$ increases the incentives to put forth effort, and it is optimal to increase both of them all the way to one. ■

Theorem 2 *Suppose that no type of contestant is confident.*

- *If* $D \leq \frac{(1+C_B)(1+C_A)}{1-C_A C_B}$*, then in any PBE both types of contestant put forth effort in both periods and the designer occasionally overturns the judge.*

- *If* $\frac{(1+C_B)(1+C_A)}{1-C_A C_B} < D \leq 1 + C_B$*, then in any PBE only contestants of type* B *put forth effort in the second period conditional on their type losing in the first. The designer sets* $\gamma_{A,1} = 1$*, and* $\gamma_{B,1}$ *to the minimum level possible so that both types put forth effort in the first period.*

- *If* $1 + C_B < D \leq 1 + C_A$*, then in any PBE only contestants of type* A *put forth effort in the second period conditional on their type losing in the first. The designer sets* $\gamma_{B,1} = 1$*, and* $\gamma_{A,1}$ *to the minimum level possible so that both types put forth effort in the first period.*

- *If* $1 + C_A < D \leq 1$*, then in any PBE neither type of contestant puts forth effort in the second period conditional on their type losing in the first. The designer sets* $\gamma_{A,1}$ *and* $\gamma_{B,1}$ *high enough so that neither contestant has the incentive to stop putting forth effort in the first period.*

- *If* $1 < D$*, then in any PBE contestants of type* A *put forth effort in both periods but contestants of type* B *do not put forth effort in either period.*

Proof. The contestants put forth maximal effort in both periods if there exist $\gamma_{A,1}$ and $\gamma_{B,1}$ which satisfy both (IC-1), (BC-A), and (BC-B). Since both C_A and C_B are negative, this is true when the intersection of the lines defined by (BC-A) and (BC-B) falls below the line defined by (IC-1). This intersection is where

$$\gamma_{A,1} - C_A \gamma_{B,1} = 1$$
$$\gamma_{B,1} - C_B \gamma_{A,1} = 1.$$

This can be solved to find that the intersection is where $\gamma_{A,1} = \frac{1+C_A}{1-C_A C_B}$ and $\gamma_{B,1} = \frac{1+C_B}{1-C_A C_B}$. This intersection satisfies (IC-1) if

$$\frac{1 + C_A}{1 - C_A C_B} + \frac{1 + C_B}{1 - C_A C_B} - 1 \geq D,$$

or $\frac{(1+C_A)(1+C_B)}{1-C_A C_B} \geq D$.

Next, we show the conditions under which there exist $\gamma_{A,1}$ and $\gamma_{B,1}$ which satisfy both (IC-1) and (BC-A). This is only possible when $C_A > -1$, because otherwise the only $\gamma_{A,1}$ and $\gamma_{B,1}$ which satisfied (BC-A) would need to sum to less than one, which violates (IC-1) for *any* v. Furthermore, when $C_A > -1$, lowering $\gamma_{A,1}$ while increasing $\gamma_{B,1}$ by the same value relaxes (BC-A) while having no effect on (IC-1), so we can focus on the case where $\gamma_{B,1} = 1$. Checking whether both (IC-1) and (BC-A) can be satisfied simultaneously is equivalent to checking whether there exists a $\gamma_{A,1}$ which satisfies

$$\gamma_{A,1} \geq D$$

and

$$\gamma_{A,1} \leq 1 + C_A.$$

Thus, it is possible to satisfy (IC-1) and (BC-A) simultaneously if $D \leq 1 + C_A$. Because the case for type B is symmetric, it is possible to satisfy (IC-1) and (BC-B) simultaneously if $D \leq 1 + C_B$.

Suppose that the designer chooses $\gamma_{A,1}$ and $\gamma_{B,1}$ such that (IC-1) and (BC-A) are satisfied, but (BC-B) is not satisfied. In this case, contestants put forth a total of two effort in both periods when type B is awarded the prize, but only one effort in the second period when type A is awarded the prize. Thus, the expected discounted sum of effort is

$$2 + \mathbb{P}(B \text{ wins}|a_{A,1} = a_{B,1} = 1)(2\delta) + \mathbb{P}(A \text{ wins}|a_{A,1} = a_{B,1} = 1)(\delta)$$

which can be expanded as

$$\begin{aligned} 2+&\left(\frac{1}{2}(1-\gamma_{A,1})(1+p_{A,1}-p_{B,1}) + \frac{1}{2}\gamma_{B,1}(1+p_{B,1}-p_{A,1})\right)(2\delta) \\ &+\left(\frac{1}{2}\gamma_{A,1}(1+p_{A,1}-p_{B,1}) + \frac{1}{2}(1-\gamma_{B,1})(1+p_{B,1}-p_{A,1})\right)(\delta). \end{aligned}$$

This simplifies to

$$2 + \left(\frac{3}{2} + \frac{1}{2}p_{A,1} - \frac{1}{2}p_{B,1}\right)\delta - \left(\frac{3}{2} + \frac{1}{2}p_{A,1} - \frac{1}{2}p_{B,1}\right)\gamma_{A,1}\delta + \left(\frac{3}{2} + \frac{1}{2}p_{B,1} - \frac{1}{2}p_{A,1}\right)\gamma_{B,1}\delta.$$

Thus, the expected discounted sum of effort is decreasing in $\gamma_{A,1}$ and increasing in $\gamma_{B,1}$. The optimal values of $\gamma_{A,1}$ and $\gamma_{B,1}$ subject to them satisfying (IC-1) and (BC-A) (but not (BC-B)) are $\gamma_{B,1} = 1$

and $\gamma_{A,1} = D$. For the optimal choice of $\gamma_{A,1}$ and $\gamma_{B,1}$, the expected discounted sum of effort is

$$2 + 3\delta - \left(\frac{3}{2} + \frac{1}{2}p_{A,1} - \frac{1}{2}p_{B,1}\right)\delta D.$$

A symmetric argument shows that when satisfying (IC-1) and (BC-B) (but not (BC-A)), the expected discounted sum of effort is

$$2 + 3\delta - \left(\frac{3}{2} + \frac{1}{2}p_{B,1} - \frac{1}{2}p_{A,1}\right)\delta D.$$

Thus, when $p_{A,1} \geq p_{B,1}$ and $\frac{(1+C_B)(1+C_A)}{1-C_A C_B} < D \leq 1 + C_B \leq 1 + C_A$, it is optimal to set $\gamma_{A,1} = 1$, $\gamma_{B,1} = D$ so that the contestant of type B puts forth effort conditional on losing but the contestant of type A does not. If $1 + C_B \leq D \leq 1 + C_A$, then it is not possible to ensure that the contestant of type B puts forth effort conditional on losing, and it is optimal to set $\gamma_{A,1} = D$ and $\gamma_{B,1} = 1$.

The proof of Theorem 1 demonstrates that there are values of $\gamma_{A,1}$ and $\gamma_{B,1}$ which satisfy (IC-1) if and only if $D \leq 1$. This implies that the designer can ensure that (IC-1) but not (BC-A) or (BC-B) can be satisfied when $1 + C_B \leq 1 + C_A < D \leq 1$. Because $p_{A,1} \geq p_{B,1}$ implies that $C_A \geq C_B$ when C_A is negative, this exhausts the cases. ∎

Theorem 3 *Suppose that contestants of type A are confident but contestants of type B are not confident.*

- *If $D \leq 1 + C_B$, then in any PBE both types of contestant put forth effort in both periods and the designer occasionally overturns the judge.*
- *If $1 + C_B < D \leq 1$, then in any PBE both types of contestant put forth effort in the first period, but only contestants of type A put forth effort in the second period conditional on their type losing in the first. The designer sets $\gamma_{A,1}$ and $\gamma_{B,1}$ high enough so that neither contestant has the incentive to stop putting forth effort in the first period.*
- *If $1 < D$, then in any PBE contestants of type A put forth effort in both periods but contestants of type B do not put forth effort in either period.*

Proof. Type A being confident implies that (BC-A) is satisfied for any feasible values of $\gamma_{A,1}$ and $\gamma_{B,1}$.

Since (BC-A) is satisfied, the contestants put forth maximal effort in both periods if there exist $\gamma_{A,1}$ and $\gamma_{B,1}$ which satisfy both (IC-1) and (BC-B). In the proof of Theorem 2 we showed that such

$\gamma_{A,1}$ and $\gamma_{B,1}$ exist if and only if $D \leq 1 + C_B$.

The proof of Theorem 1 demonstrates that there are values of $\gamma_{A,1}$ and $\gamma_{B,1}$ which satisfy (IC-1) if and only if $D \leq 1$. This implies that the designer can ensure that (IC-1) but not (BC-B) can be satisfied when $1 + C_B < D \leq 1$. ■

Theorem 4 *Suppose that both types of contestant are confident. Then both types put forth effort in both periods. The designer sets $\gamma_{A,1}$ and $\gamma_{B,1}$ high enough so that neither contestant has the incentive to stop putting forth effort in the first period.*

Proof. Both players being confident implies that (BC-A) and (BC-B) are both satisfied when $\gamma_{A,1} = \gamma_{B,1} = 1$. Furthermore, when type A is confident,

$$2p_{B,1} + \left(\frac{2}{v} - 1\right)(1 + p_{B,1} - p_{A,1}) \leq 0$$

which can be restated as

$$\begin{aligned}\frac{2}{v}\left(\frac{1}{1 + p_{A,1} - p_{B,1}}\right) &\leq \frac{1 - p_{A,1} - p_{B,1}}{1 - (p_{A,1} - p_{B,1})^2} \\ &\leq 1.\end{aligned}$$

which is exactly (IC-1). Thus, for $\gamma_{A,1} = \gamma_{B,1} = 1$, (IC-1), (BC-A), and (BC-B) are all satisfied and both types of contestant put forth effort in both periods. ■

Proposition 1 *In the equilibrium of the single period model with screening and $\lambda < 1$, both contestants put forth effort if*

$$D(v, p_{A,1}, p_{B,1}) \leq 1.$$

Otherwise, only type A puts forth effort. The designer sets $\gamma_{A,1}$ and $\gamma_{B,1}$ equal to 1 if both types are putting forth effort.

Proof. This result follows from the proof of Theorem 1 and the discussion in the text. ■

Proposition 2 *In the equilibrium of the two period model with screening and $\lambda < 1$, the designer maximizes her payoffs by setting $\gamma_{i,t} = 1$ for all i and t if (1) neither type is confident and $1+C_A < D$, (2) type A is confident and $1 < D$, or (3) both types are confident.*

Proof. The proofs of Theorem 2, Theorem 3, and Theorem 4 showed that under the conditions given in the statement of the proposition, lowering $\gamma_{i,t}$ for any i or t below 1 weakly decreases effort from both types and in both periods.

If both types put forth effort in a period then the likelihood that the prize is awarded to a high type in any period is

$$\mu^2 + \mu(1-\mu)[(1-p_{B,t})\gamma_{A,t} + p_{B,t}(1-\gamma_{B,t})] + \mu(1-\mu)[(1-p_{A,t})\gamma_{B,t} + p_{A,t}(1-\gamma_{A,t})]$$
$$= \mu^2 + \mu(1-\mu)[(\gamma_{A,t}+\gamma_{B,t})(1-p_{A,t}-p_{B,t}) + p_{A,t} + p_{B,t}].$$

This is strictly decreasing in $\gamma_{A,t}$ and $\gamma_{B,t}$, so the designer is strictly better off setting both values equal to one.

If only one type is putting forth effort in period t, then the likelihood that the prize is awarded to a high type is μ regardless of whether the judge is biased or the values of $\gamma_{A,t}$ or $\gamma_{B,t}$. ■

Proposition 3 *Suppose that neither type is confident and* $D \leq \frac{(1+C_B)(1+C_A)}{1-C_A C_B}$. *Then if*

$$\delta \geq \frac{(1-\lambda)(1-\mu)\mu(1-p_{i,1}-p_{j,1})(2C_A C_B + C_A + C_B)}{[\lambda + (1-\lambda)(\mu^2 + \mu(1-\mu)(2-p_{i,1}-p_{j,1}) - \mu)](C_A C_B - 1)},$$

committing to occasionally overturn the judge increases the designer's payoffs.

Proof. The proof of Theorem 2 shows that when neither type is confident and $D \leq \frac{(1+C_B)(1+C_A)}{1-C_A C_B}$, neither type puts forth effort in the second period conditional on their type losing in the first period. It also shows that there exists $\gamma_{A,1}$ and $\gamma_{B,1}$ that induces both types to put forth effort in both periods.

Suppose that the conditions of the proposition hold, and that $\gamma_{A,1}$ and $\gamma_{B,1}$ satisfy (IC-1), (BC-A), and (BC-B). Then the payoffs to the designer in the second period are

$$\lambda(1+1) + (1-\lambda)[\mu^2 + \mu(1-\mu)(1-p_{A,1}) + \mu(1-\mu)(1-p_{B,1})]$$
$$= 2\lambda + (1-\lambda)[\mu^2 + \mu(1-\mu)(2-p_{A,1}-p_{B,1})]$$

while the payoffs in the first period are

$$2\lambda + (1-\lambda)[\mu^2 + \mu(1-\mu)[(1-p_{B,1})\gamma_{A,1} + p_{B,1}(1-\gamma_{B,1})] + \mu(1-\mu)[(1-p_{A,1})\gamma_{B,1} + p_{A,1}(1-\gamma_{A,1})]]$$
$$= 2\lambda + (1-\lambda)[\mu^2 + \mu(1-\mu)[(\gamma_{A,1}+\gamma_{B,1})(1-p_{A,1}-p_{B,1}) + p_{A,1} + p_{B,1}]].$$

Notice that these first period payoffs are strictly decreasing in $\gamma_{A,1} + \gamma_{B,1}$. This sum is maximized by choosing $\gamma_{A,1}$ and $\gamma_{B,1}$ which satisfy (BC-A), and (BC-B) with equality, so

$$\gamma_{A,1} = \frac{1+C_A}{1-C_AC_B}$$
$$\gamma_{B,1} = \frac{1+C_B}{1-C_AC_B}$$

and the payoffs from the first period are

$$2\lambda + (1-\lambda)\left[\mu^2 + \mu(1-\mu)\left[\left(\frac{2+C_A+C_B}{1-C_AC_B}\right)(1-p_{A,1}-p_{B,1}) + p_{A,1}+p_{B,1}\right]\right]$$

If instead the designer is setting $\gamma_{A,1} = \gamma_{B,1} = 1$, then she receives $\lambda + (1-\lambda)\mu$ in the second period and $2\lambda + (1-\lambda)[\mu^2 + \mu(1-\mu)(2-p_{i,1}-p_{j,1})]$ in the first period.

Thus, setting $\gamma_{A,1}$ and $\gamma_{B,1}$ to satisfy (IC-1), (BC-A), and (BC-B) is optimal only if

$$\begin{aligned}
&2\lambda + (1-\lambda)\left[\mu^2 + \mu(1-\mu)\left[\left(\frac{2+C_A+C_B}{1-C_AC_B}\right)(1-p_{A,1}-p_{B,1}) + p_{A,1}+p_{B,1}\right]\right] \\
&\quad + \delta\left[2\lambda + (1-\lambda)[\mu^2 + \mu(1-\mu)(2-p_{A,1}-p_{B,1})]\right] \\
\geq &2\lambda + (1-\lambda)[\mu^2 + \mu(1-\mu)(2-p_{A,1}-p_{B,1})] \\
&\quad + \delta[\lambda + (1-\lambda)\mu]
\end{aligned}$$

which can be rewritten as

$$\begin{aligned}
&\delta\left[\lambda + (1-\lambda)(\mu^2 + \mu(1-\mu)(2-p_{A,1}-p_{B,1}) - \mu)\right] \\
&\quad \geq (1-\lambda)(1-\mu)\mu(1-p_{A,1}-p_{B,1})\left(\frac{C_A+C_B+2C_AC_B}{C_AC_B-1}\right).
\end{aligned}$$

Thus, we know that committing to overturn the judge gives higher payoffs than always respecting the judge's ranking if

$$\delta \geq \frac{(1-\lambda)(1-\mu)\mu(1-p_{A,1}-p_{B,1})(2C_AC_B+C_A+C_B)}{[\lambda + (1-\lambda)(\mu^2 + \mu(1-\mu)(2-p_{A,1}-p_{B,1}) - \mu)](C_AC_B-1)}$$

which is the condition provided in the proposition. ■

Corollary 1 *Suppose that neither type is confident, $\lambda = 0$, and $D \leq \frac{(1+C_B)(1+C_A)}{1-C_AC_B}$. Then there exists a $\delta^* \in (0,1)$ such that if $\delta > \delta^*$, committing to occasionally overturn the judge increases the designer's payoffs.*

Proof. Notice that if we have $\lambda = 0$, then the condition from Proposition 3 simplifies to

$$\begin{aligned}
\delta &\geq \frac{(1-\mu)\mu(1-p_{i,1}-p_{j,1})(2C_AC_B+C_A+C_B)}{[(\mu^2+\mu(1-\mu)(2-p_{i,1}-p_{j,1})-\mu)](C_AC_B-1)} \\
&= \frac{(1-\mu)(1-p_{i,1}-p_{j,1})(2C_AC_B+C_A+C_B)}{[(\mu+(1-\mu)(2-p_{i,1}-p_{j,1})-1)](C_AC_B-1)} \\
&= \frac{(1-p_{i,1}-p_{j,1})(2C_AC_B+C_A+C_B)}{[((2-p_{i,1}-p_{j,1})-1)](C_AC_B-1)} \\
&= \frac{2C_AC_B+C_A+C_B}{C_AC_B-1} \\
&= \frac{(1+C_A)(1+C_B)}{C_AC_B-1}+1.
\end{aligned}$$

Because $-1 < C_B \leq C_A < 0$ when neither type is confident and $D \leq \frac{(1+C_B)(1+C_A)}{1-C_AC_B}$, we know that $-1 < \frac{(1+C_A)(1+C_B)}{C_AC_B-1} < 0$. Thus, if δ is close enough to one, the inequality from Proposition 3 will be satisfied. ■